Mathematical Modeling: Math 420/620
Modeling Forest Population Response to Climate Change, HW B

Problem B. Reconsider the Sample Problem 2, in which we assumed that the rates at which hardwood population competes within species and with softwood population were increasing in the next few hundred years. The governing system of the equations again is

\[
\begin{align*}
\frac{dH}{dt} &= 0.10H - \frac{0.10 + 10^{-3}t}{10000} H^2 - \frac{0.05 + 10^{-3}t}{10000} HS \\
\frac{dS}{dt} &= 0.25S - \frac{0.25}{6000} S^2 - \frac{0.25}{12000} HS.
\end{align*}
\]

We analyzed the location of critical points at a few time instances.

(a) How sensitive is the location of the coexistence critical point at \( t = 0 \) and \( t = 100 \) to the coefficient \( 10^{-3} \) in the \( H^2 \) term? To the coefficient \( 10^{-3} \) in the \( HS \) term?

(b) At what time will the coexistence equilibrium no longer exist? Which species will die out, and when will it occur? Provide some graphical description of how the equilibrium population varies as a function of time.

(c) Suppose that there are currently 100 tons/acre of hardwoods and 1,500 tons/acre of softwood trees in the forest. What does this model predict about the future of the two species over the next several hundred years?

(d) Comment on how the results will change if \( 10^{-3} \) coefficient in \( H^2 \) term is replaced by \( 10^{-2} \), by \( 10^{-4} \).