Math 373 – Review sheet

Sun, 24 Oct 2010 / A Kumjian

Test II: Thurs, 28 Oct

Review: Tues, 26 Oct

Test I will cover sections 3.1 – 3.4, 4.1 – 4.5, 5.1 – 5.5, 6.1. You may bring a formula sheet to the test; it may include definitions, results, propositions etc., but please do not include sample proofs or other worked out problems. There will be a supplementary office hour Wednesday afternoon 4-5 pm.

To receive full credit, solutions must be correct, complete and neatly written; give reasons for your answers. Review the text and all assigned homework problems (and their solutions). Also review the solutions to quizzes 4, 5, 6, 7. The quiz problems are listed below:

1. Let $x \in \mathbb{Z}$. Prove that if $3x + 7$ is odd, then x is even.
2. Let $n \in \mathbb{Z}$. Prove that $n^2 + n + 3$ is odd.
3. Let $n \in \mathbb{Z}$. Prove that $3 \mid (n^2 + 2)$ implies $3 \nmid n$.
4. Let $A = \{n \in \mathbb{Z} : n \equiv 3 \pmod 5\}$ and $B = \{n \in \mathbb{Z} : n^2 \equiv 4 \pmod 5\}$. Prove that $A \subseteq B$.
5. Disprove the statement: For every $x \in \mathbb{R}$, $x^2 + 3x + 2 \geq 0$
6. Prove that $\sqrt{5}$ is irrational. [Hint: You may use the fact: $\forall n \in \mathbb{Z}$, $5 \mid n$ if and only if $5 \mid n^2$.]
7. Use the Intermediate Value Theorem to prove that there exists $x \in (0, 2)$ such that
$$x^3 - 2x^2 + 3x = 4.$$
8. Use mathematical induction to prove that, for every positive integer n,
$$3 + 7 + \cdots + (4n - 1) = n(2n + 1).$$

Here are some other sample problems:

9. Let $a, b, n \in \mathbb{Z}$ and suppose that $n \geq 2$. Recall (see Ex. 4.2) that $a \equiv b \pmod n$ implies $a^2 \equiv b^2 \pmod n$. Is the converse true?
10. Let $n \in \mathbb{N}$. Prove that if $11 \nmid n$, then there is $k \in \{1, 3, 4, 5, 9\}$ such that $n^2 \equiv k \pmod{11}$. Prove that $11 \mid n$ if and only if $11 \mid n^2$.
11. Use the results of the preceding problem to prove that $\sqrt{11}$ is irrational.
12. Prove that there is a positive integer n such that
$$\frac{4n^2 + 35}{12n} < 2.$$
13. Disprove the following statement: There is a real number x such that $x^2 + 5 = 4x$.
14. Use mathematical induction to prove that for every positive integer n,
$$3 \mid (4^n - 1).$$
15. Use mathematical induction to prove that for every $n \in \mathbb{N}$,
$$n^2 < 2^{n+1}.$$
16. Let $a, b, c, d, n \in \mathbb{Z}$ and suppose that $n \geq 2$. Suppose $bc \equiv d \pmod n$ and $ab \equiv 1 \pmod n$. Use Result 4.11 to prove that $c \equiv ad \pmod n$. [Hint: Exercise 4.11 might also be useful.]

In addition, please review the exercises:
3.19, 3.25, 3.32, 3.38, 3.39, 3.43, 4.5, 4.11, 4.15, 4.21, 4.25, 4.27, 4.49, 4.50, 5.3, 5.7, 5.18a, 5.23, 5.33, 5.35, 5.39, 6.39.