Theoretical overview of atomic parity violation

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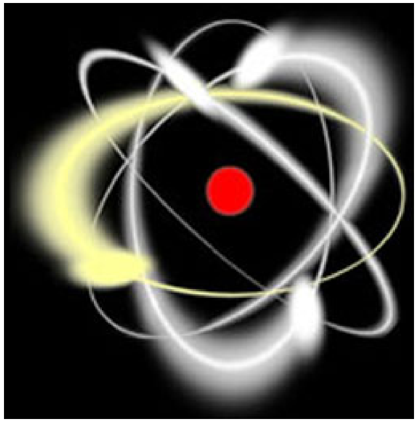
Talk at PAVI workshop, Milos, Greece May 2006



Outline
Atomic Parity Violation (APV)

- How does APV work?
- Historical notes
- Single-isotope measurements (focus on Cs)
  - Review of recent theoretical developments: Breit, radiative, and neutron skin
  - Updated value of the weak charge
  - Challenges
- APV measurements in a chain of isotopes
  - Neutron skin/halo complications

Collaborators

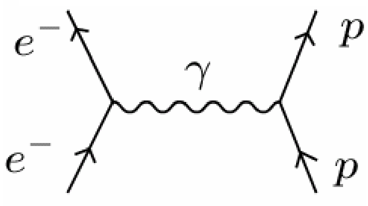
UNR Theoretical Atomic Physics group with visitors, Basque restaurant, Apr 2006

Atomic parity violation

Parity transformation: $\mathbf{r}_i \rightarrow -\mathbf{r}_i$

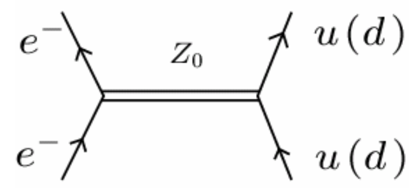
$[H_{atomic}, P]=0$ => Atomic stationary states are eigenstates of Parity

Electromagnetic



Conserve parity

Electroweak



Do not conserve parity

Z-boson exchange spoils parity conservation

What is the strength of electroweak coupling of quarks and electrons?

Historical notes

- Zel'dovich 1958: APV idea; optical activity; effect too small?
- Realization that P-violation may be observable in heavy atoms (Bouchiat & Bouchiat 1974)
- First observation: optical rotation in ^{209}Bi (Barkov & Zolotorev 1978)
- Other observations: Tl, Pb, Cs atoms
- Most accurate (0.35%) experiment in ^{133}Cs (Colorado group 1997); discovery of the anapole moment

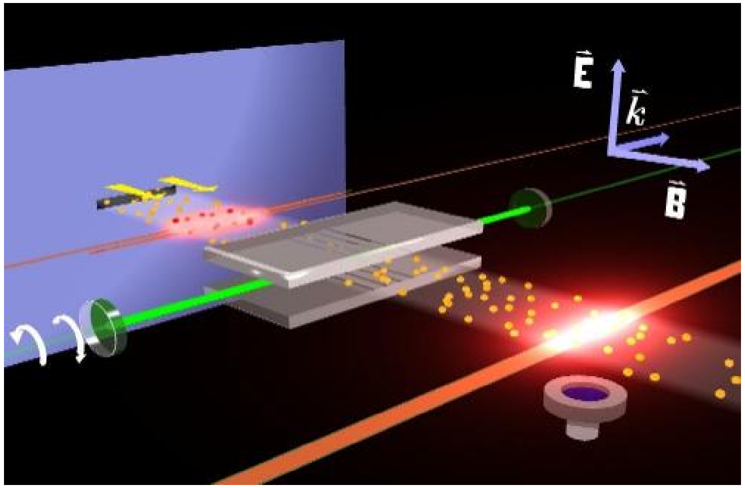


Table-top APV experiment using Stark-interference by Colorado group

Talks on Atomic Parity Violation @ PAVI'06

16-MAY-2006

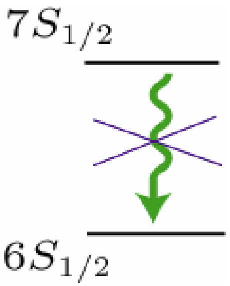
19:15 – 19:45 (30 min)	Michel Lintz	Kastler Brossel Lab	A Precise PV Measurement in the Cesium Atom using Stimulated Emission

17-MAY-2006 Morning Session

10:45 – 11:15 (30 min)	Christian Chardonnet	University of Paris XIII	Parity Violation in Molecules: Towards a First Experimental Observation
11:15 – 11:45 (30 min)	Dmitry Budker	University of California at Berkeley	Probing the Isotopic Dependence of APV in Ytterbium
11:45 – 12:05 (20 min)	K. Tsigutkin	University of California at Berkeley	Parity Nonconservation in Atomic Ytterbium: Status of the Berkeley Experiments
12:05 – 12:35 (30 min)	Gerald Gwinner	University of Manitoba	Atomic Parity Nonconservation Measurements in Francium
12:35 – 12:55 (20 min)	Stefano Sanguinetti	University of Pisa	The TRAP-RAD Experiment: Prospects for Parity Violation Measurements in Cold Francium Atoms

Not presented here Ba+ (Seattle)

Parity-violating 7S-6S Amplitude in Cs



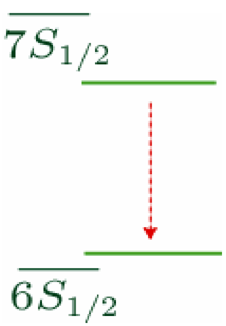
$$\langle 7S_{1/2} | D | 6S_{1/2} \rangle \equiv 0 \qquad D = \sum_{i=1}^{N} -e\,\mathbf{r}_i$$

Electric-dipole transition is forbidden by the **parity** selection rules

Weak interaction leads to an admixture of states of opposite parity (H_W is a pseudoscalar)

$$\left|\overline{6S_{1/2}}\right\rangle = |6S_{1/2}\rangle + \sum_m |mP_{1/2}\rangle \frac{\langle mP_{1/2}|H_W|6S_{1/2}\rangle}{E_{6S} - E_{mP_{1/2}}}$$

Similarly for $\left|\overline{7S_{1/2}}\right\rangle$



$$E_{\mathrm{PV}} = \left\langle \overline{7S_{1/2}} \right| D \left| \overline{6S_{1/2}} \right\rangle =$$

$$\sum_m \frac{\langle 7S_{1/2}|D|mP_{1/2}\rangle\langle mP_{1/2}|H_W|6S_{1/2}\rangle}{E_{6S} - E_{mP_{1/2}}} + \mathrm{c.c.}(6S \leftrightarrow 7S)$$

Tiny effect $\quad E_{\mathrm{PV}} \sim 10^{-11}$ atomic units

Weak charge extraction

Electron-quark PV interaction (exchange of virtual Z^0 boson)

$$H_W = \frac{G_F}{\sqrt{2}}\left(\bar{e}\gamma_\mu\gamma_5 e\right)\left\{C_{1u}\bar{u}\gamma^\mu u + C_{1d}\bar{d}\gamma^\mu d\right\} + \ldots$$

In electronic sector

$$H_W = Q_W \times \frac{G_F}{\sqrt{8}}\gamma_5\,\rho_n\left(r\right)$$

Weak charge

neutron distribution

$$Q_W^{tree} = -N + Z\left(1 - 4\sin^2\theta_W\right) \approx -N$$

PV signal

$$E_{PV} = k_{PV}\, Q_W^{\text{inferred}}$$

measured

atomic-structure calculations

Weak charge of ^{133}Cs (as of 1999)

Atomic Experiment E_{PV}

Atomic Structure Theory E_{PV} / Q_W

$$\Rightarrow Q_W^{\text{inferred}} = -72.06(28)_{\text{expt}}(34)_{\text{theor}}$$

Standard Model $Q_W^{\text{SM}} = -73.09(3)$

$$Q_W^{\text{inferred}} \neq Q_W^{\text{SM}}$$

2.5σ deviation (??? new physics, other corrections ???)

New physics scenarios:

extra Z-bosons, scalar leptoquarks, four-fermion contact interactions, etc

Experiment: Wood *et al*. (1997); Bennett and Wieman (1999) (Boulder group)
Theory: Dzuba, Sushkov, Flambaum (1989); Blundell, Johnson, and Sapirstein (1990).
SM calculations: Marciano and Rosner PRL (1990); Groom *et al* Eur. Phys. J (2000)

Deviation from the Standard Model in PV with ^{133}Cs (May 2006)

$$\sigma = 0.53\% \qquad (\sigma_{expt} = 0.35\%, \sigma_{theor} = 0.4\%)$$

1999 Based on decade-old calculations by Dzuba *et al.* and Blundell *et al.*	2.5σ	Bennett & Wieman 1999
Breit interaction	-1.2σ	Derevianko (2000) , Dzuba *et al* (2001), Kozlov *et al* (2001); Shabaev *et al.* (2005)
Vacuum polarization (+ 0.8 σ) Vertex/self-energy (-1.3 σ)	-0.5σ	Johnson *et al.* (2002);Milstein & Sushkov (2002);Kuchiev & Flambaum (2002);Sapirstein *et al.* (2003);Shabaev *et al.* (2005)
Neutron skin	-0.4σ	Derevianko (2002)
Updated correlated value and vec. trans. polarizability β	$+0.7\sigma$	Dzuba, Flambaum & Ginges (2002)
PV e-e, renormalization q->0, virtual exc. of the giant nuc. res.	$-0.08\ \sigma$	Sushkov & Flambaum (1978) Milstein,Sushkov&Terekhov (2002)
Total deviation	1.0σ	

High-precision atomic calculations

- Weak interaction occurs in the nucleus $v/c \sim \alpha Z \approx 0.4$ for Cs

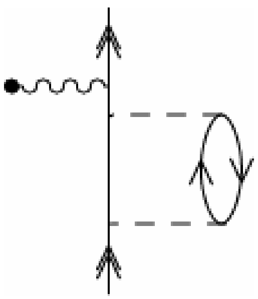
  Dirac equation as a starting point

- sub-1% accuracy is a must

  Hartree-Fock calculations are off by 50% for hyperfine-structure constant of Cs ground state

  Many-body perturbation theory in residual e-e interaction (correlations)

Most accurate MBPT calculations with reevaluated error bars



$$E_{\mathrm{PNC}} = \begin{cases} 0.908(5) & \text{Dzuba, Sushkov, Flambaum (1989)} \\ 0.909(4) & \text{Blundell, Johnson, and Sapirstein (1990)} \end{cases}$$

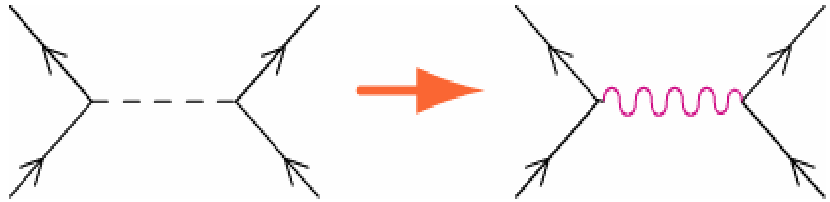
units $i|e|a_B\left(\frac{Q_W}{-N}\right)\times 10^{-11}$

Uncertainty is estimated by comparing with high-accuracy expt. atomic data

Smaller corrections (editorial remarks)

- Correlations (contributions in the e-e Coulomb interactions) 20%
  - Difficult many-body problem
- Smaller (< 1%) corrections
  - Easier, yet non-trivial set of problems
    - Breit
    - radiative
    - neutron skin
  - Substantial progress over the last 5 years

Breit interaction



instantaneous Coulomb retardation + magnetic interaction

- Breit is small – but apparent solution, lowest-order PT in *B* is not adequate!
- HF equations are solved self-consistently –
  Breit modifies HF orbitals, i.e. many-body basis
  this effect propagates through the entire MBPT calculations

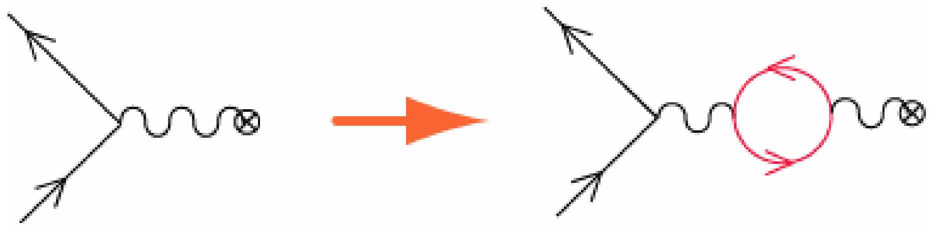
Breit corrections to PNC amplitude

Blundell *et al* (1990); HF	0.002
Derevianko (2000)	0.0054
Dzuba *et al* (2001)	0.0053

⇒ -0.6% correction, accounts for the dominant part of the deviation with the SM

Sushkov (2001) – radiative corrections should be comparable to Breit

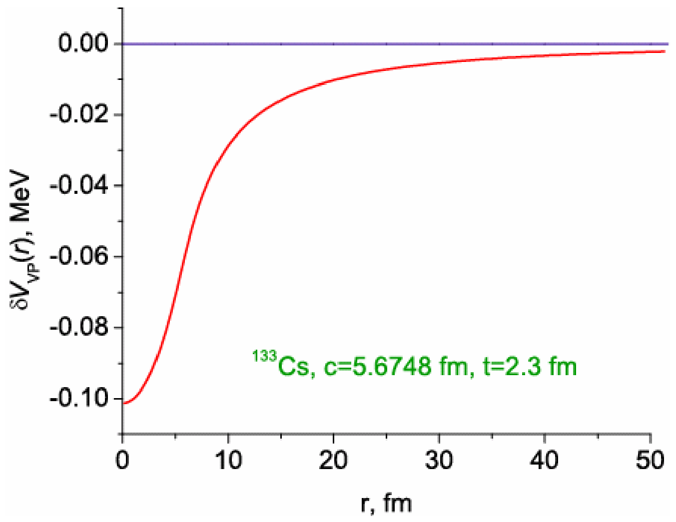
Vacuum polarization



Uehling potential

$$\delta V_{VP}(r) = -\frac{2}{3\pi}\frac{\alpha Z}{r}\int_1^\infty dt\sqrt{t^2-1}\left(\frac{1}{t^2}+\frac{1}{2t^4}\right)\exp[-2ctr]$$

$$-\frac{Z}{r} \Rightarrow -\frac{Z}{r} + \delta V_{VP}(r)$$



- VP modifies E_{PV} by 0.4%
- partially cancels –0.6% Breit correction
- increases the deviation of APV Q_W from the SM

Johnson, Bednyakov and Soff, PRL (2001)

Radiative Binding corrections

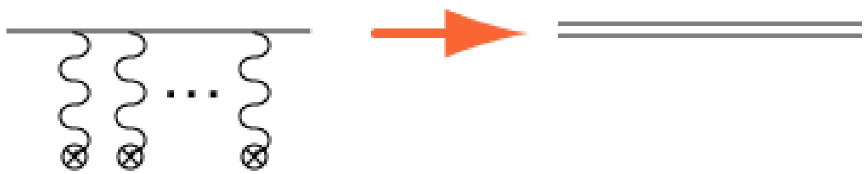
Radiative corrections to APV in Standard Model:

Marciano and Sirlin, Phys. Rev. D (1983)

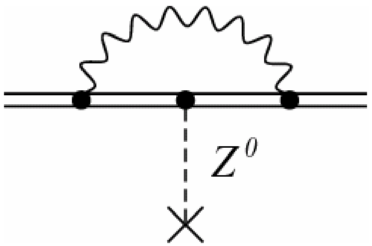
Lynn and Sandars, J.Phys. B (1994)

Free-particle propagators were used

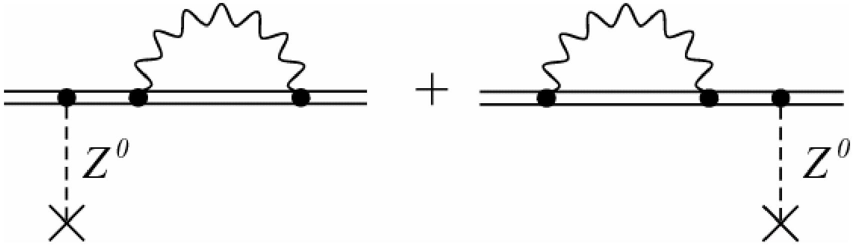
Interactions with the field of the nucleus
Binding corrections *(α Z)*



1. Vertex correction



2, Self-energy correction



Radiative binding corrections

PNC matrix element

$$\frac{\delta Q_W}{Q_W} \cong \frac{\delta \langle s_{1/2} | h_W | p_{1/2} \rangle}{\langle s_{1/2} | h_W | p_{1/2} \rangle} = \frac{\alpha}{\pi}\left[-\frac{1}{2} - \pi \left(2\ln 2 + \frac{7}{12} \right) (\alpha Z) + \dots \right]$$

Schwinger term -0.12%

Binding correction -0.57%

Z=55, αZ=0.4

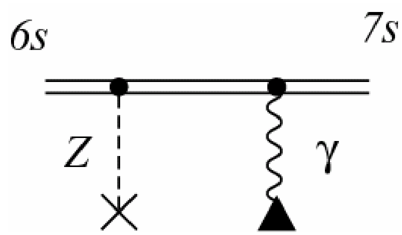
Leading in αZ	- 0.69%	Milstein, Sushkov & Terekhov, PRL (2002)
All-order in αZ	- 0.93%	J. Sapirstein *et al.*, hep-ph/0302202
From finite nuclear size and “Chiral identity”	-1.0(1)%	Kuchiev & Flambaum, PRL (2002)

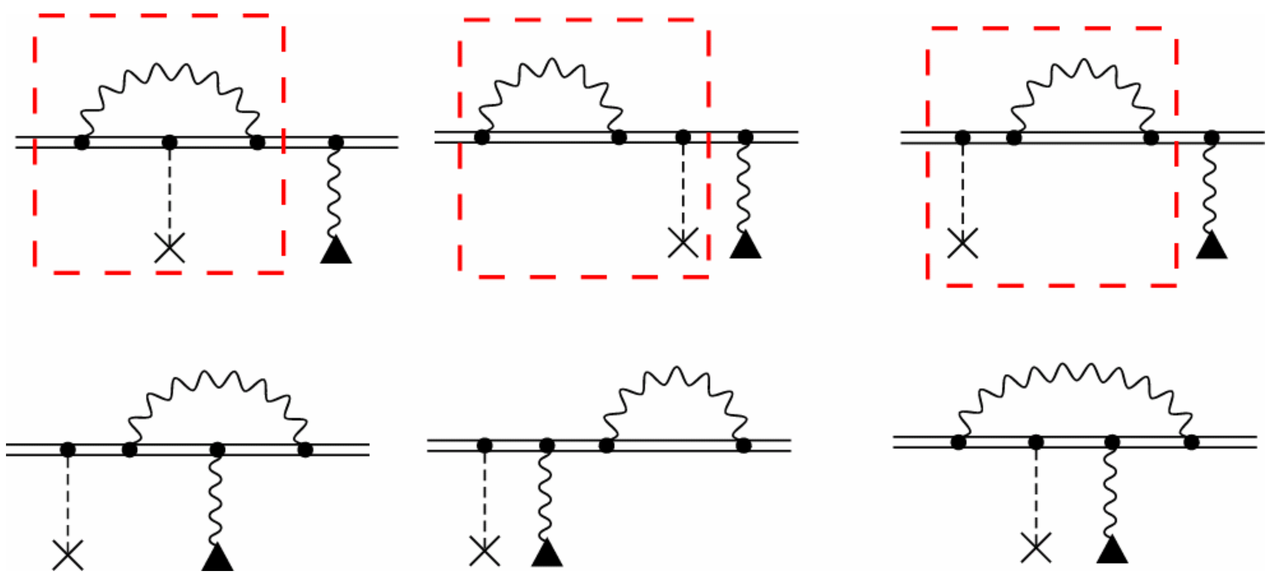
Rigorous treatment of rad. corrections (Sapirstein *et al.* formulation)

Problems:

- ❖Gauge dependence
- ❖Hydrogen-like wavefunctions
- ❖No radiative corrections to energies and dipole matrix elements

Actual gauge-independent process:





J. Sapirstein *et al.*, hep-ph/0302202

Radiative corrections (rigorous approach)

Shabaev, Tupitsyn, Pachuki, Plunien & Yerokhin PRL (2005)

- Full set of gauge independent diagrams
- Correlated (model-potential) wavefuncions

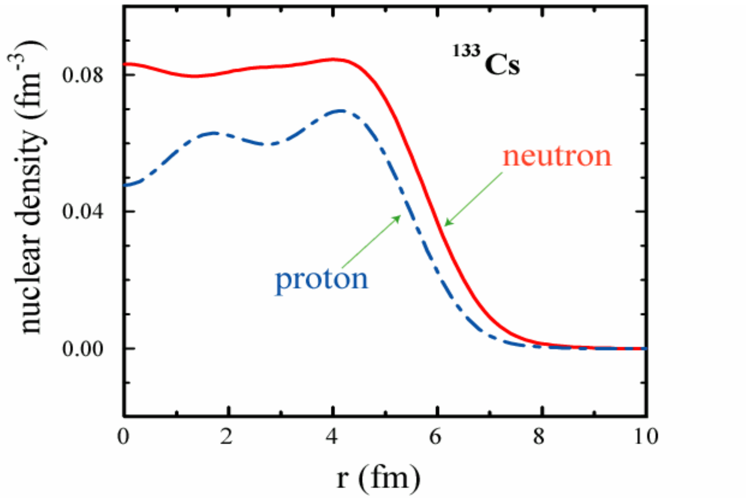
TABLE II. The SE corrections to the $6s$-$7s$ PNC amplitude in ^{133}Cs, in percent. The results are presented in both the length (L) and the velocity (V) gauge.

Contr.	L gauge	V gauge	Contribution	L gauge	V gauge
$\delta E^{\rm a}_{\rm PNC}$	−0.09	−0.11	$\delta E^{\rm h}_{\rm PNC}$	−4.04	−3.40
$\delta E^{\rm b}_{\rm PNC}$	1.31	1.11	$\delta E^{\rm i}_{\rm PNC}$	−4.61	−3.97
$\delta E^{\rm c}_{\rm PNC}$	0.34	0.40	$\delta E^{\rm j}_{\rm PNC}$	1.49	1.73
$\delta E^{\rm d}_{\rm PNC}$	−0.38	−0.32	$\delta E^{\rm k}_{\rm PNC}$	−0.79	−1.03
$\delta E^{\rm e}_{\rm PNC}$	−1.29	−1.53	$\delta E^{\rm l}_{\rm PNC}$	2.05	1.41
$\delta E^{\rm f}_{\rm PNC}$	3.89	3.25	$\delta E^{\rm add}_{\rm PNC}$	0.00	0.10
$\delta E^{\rm g}_{\rm PNC}$	1.33	1.57	$\delta E^{\rm tot}_{\rm PNC}$	−0.79	−0.79
			$\delta E^{\rm bind}_{\rm PNC}$	−0.67	−0.67

Neutron skin/halo correction

$$H_W \approx \frac{G_F}{\sqrt{8}} \gamma_5 N \rho_{\text{neutron}}(r)$$

Well-known proton distributions are used instead => corrections needed



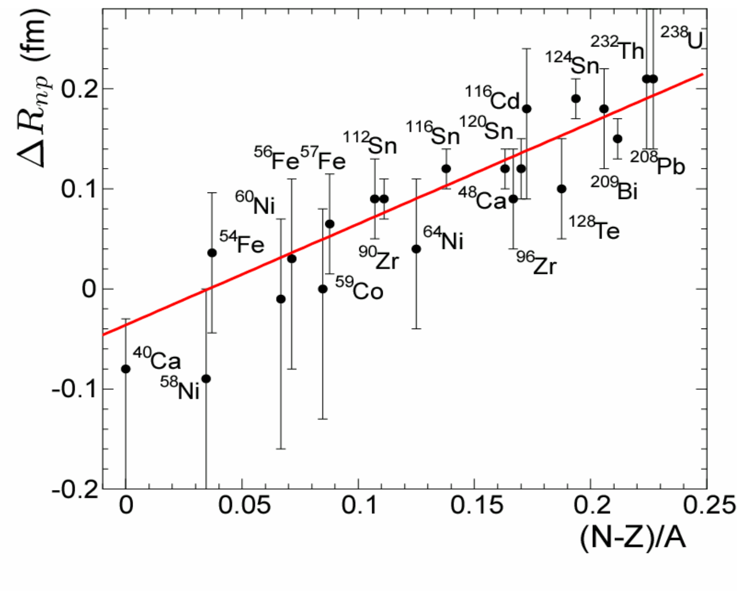
From Vretenar, Lalazissis and Ring PRC, **62** 045502 (2000)

Nuclear-structure calculations differ by a factor of four for $\Delta R_{np} = R_n - R_p$

Corrections to Q_W: $0.2\,\sigma - 0.8\,\sigma$

Nucl. Str. calcs: Pollock & Welliver, PLB, **464** 177 (1999); Vretenar, Lalazissis and Ring PRC, **62** 045502 (2000); Panda & Das PRC 62, 065501 (2000).

Neutron skin/halo correction



Experiments with anti-protonic atoms

$$\Delta R_{np} = (-0.04 \pm 0.03) + (1.01 \pm 0.15)\frac{N-Z}{A}\ \text{fm}$$

From Trzcinska *et al.*, PRL **87** 082501 (2001)

For ^{133}Cs $\quad \Delta R_{np} \approx 0.13(4)\text{fm}$

Fortson, Pang, and Willets, PRL **65** 2875 (1990)

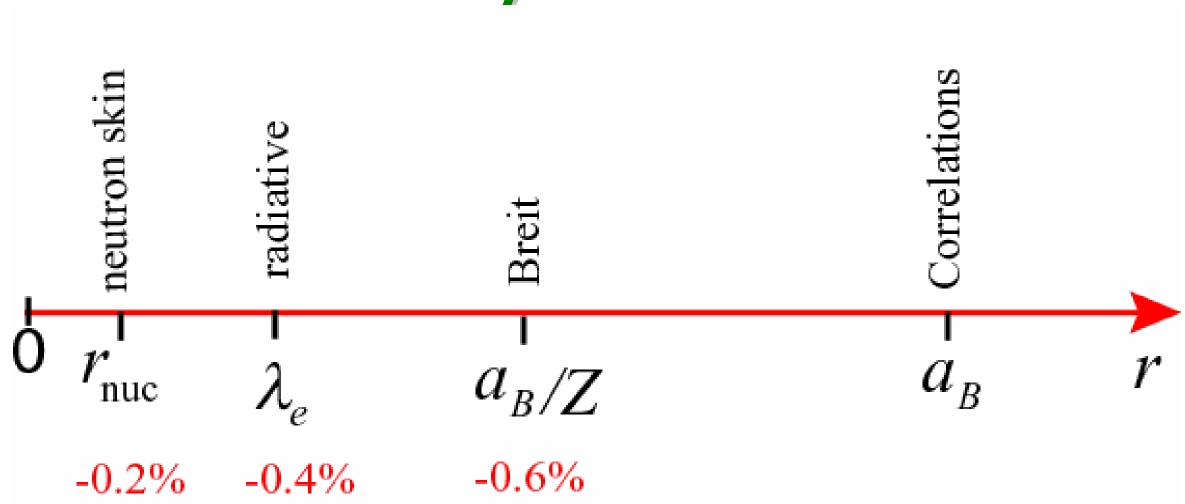
$$\frac{\delta E_{PV}^{n.s.}}{E_{PV}} \approx -\frac{3}{7}(\alpha Z)^2 \frac{\Delta R_{np}}{R_p} \approx -0.0019(6), \qquad \text{i.e.} \quad -0.2\% \quad (-0.4\sigma)$$

Error bar of 30%

AD *Phys. Rev. A* ***65****, 012106 (2002)*

Summary Cs APV

2.5 σ
+1.3%



$$\left.\begin{array}{ll}\text{Atomic Experiment} & E_{\text{PV}} \\ \text{Atomic Structure Theory} & E_{\text{PV}}/Q_W\end{array}\right\} \Rightarrow Q_W^{\text{inferred}} = -72.65(28)_{\text{expt}}(36)_{\text{theor}}$$

$$\text{Standard Model} \qquad Q_W = -73.09(3)$$

- **Theory of "smaller corrections" is settled**
- **Agreement with the Standard Model @ 1** σ level
- **? Next level of accuracy**

Thanks: V. Dzuba, V. Flambaum, W. Johnson, M. Kozlov, S. Porsev, J. Sapirstein

$$\sigma_Q = \sqrt{(\sigma_{\text{expt}})^2 + (\sigma_{\text{theor}})^2}$$

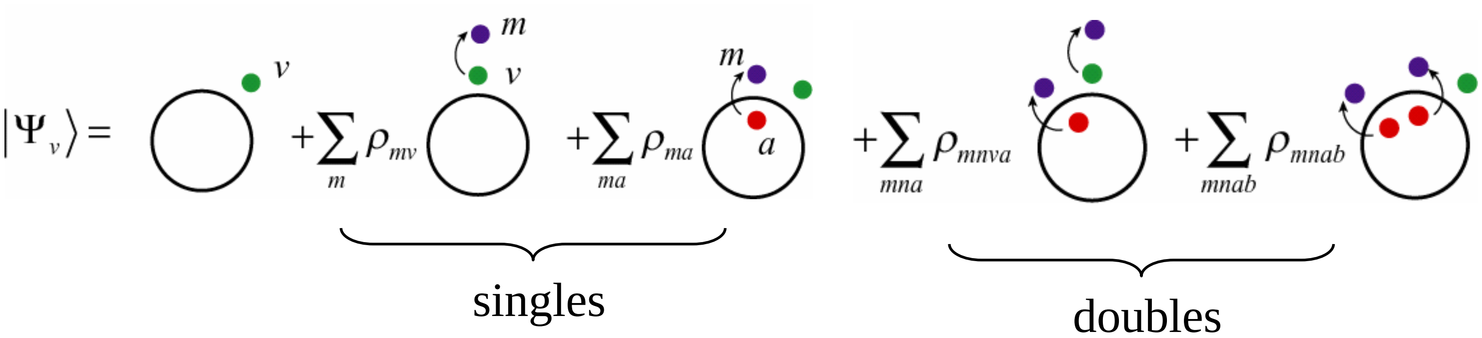
$$\sigma_{\text{expt}} = 0.35\% < \sigma_{\text{theor}} = 0.5\%$$

How to reduce σ?

Theoretical uncertainty is limited by
an accuracy of solving
the basic correlation atomic-structure problem

Tl atom (Fortson, Seattle):	measurement	1%
	theory	2.7%

Coupled-cluster method



+ triples + quadruples + …. + 55-fold excitations for Cs

$$H_{\text{atomic}} |\Psi_v\rangle = E_v |\Psi_v\rangle \quad \Rightarrow \quad \{\rho_{mv}, \rho_{ma}, \rho_{mnva}, \rho_{mnab}, \ldots\}$$

Singles-Doubles method:

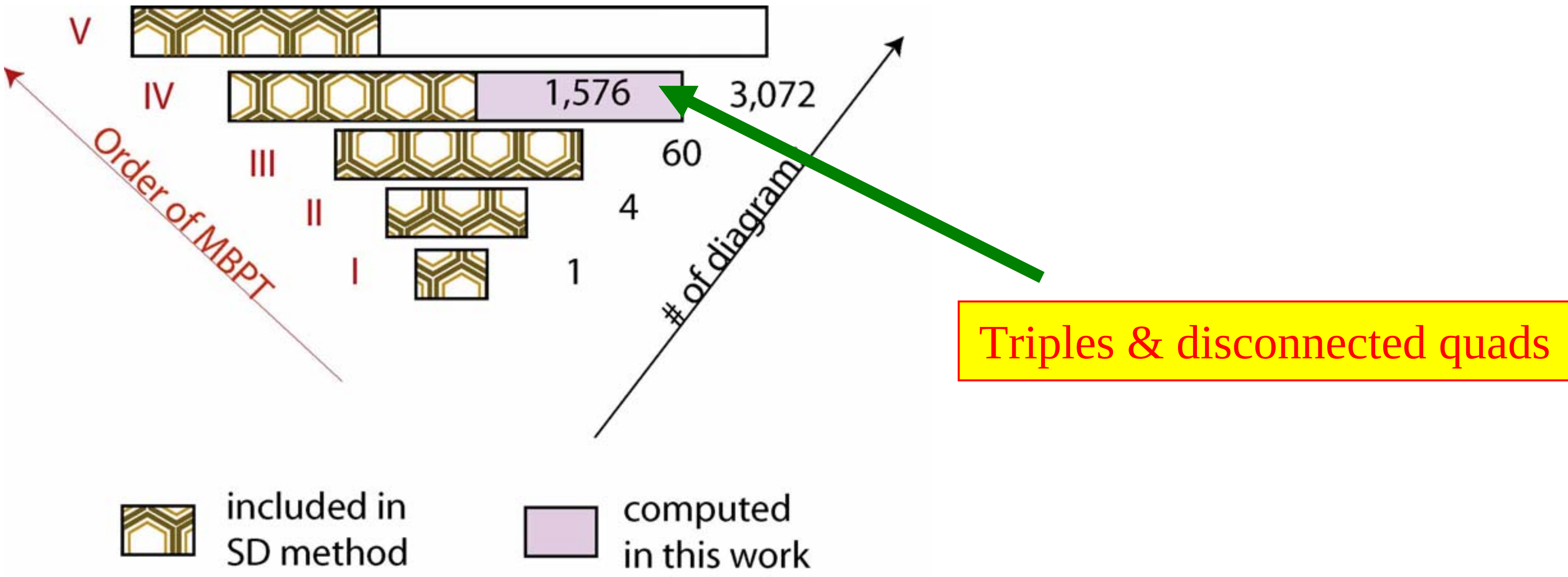
Parity violating amplitude in Cs is ~ 1% (Blundell, Johnson, Sapirstein 1990)

Large scale calculations.

Triple and higher-rank excitations are missing from the exact wavefunction. =>

Next systematic step – include triples.

Complementary IVth order diagrams



- Automated (*Mathematica*) derivation, simplification, and *Fortran90* coding.
- Even the LaTeX formulae for our papers were generated automatically

Derevianko & Emmons, Phys. Rev. A ***66****, 012503 (2002)*
Cannon & Derevianko, Phys. Rev. A ***69****, 030502(R) (2004)*

Cs calculations

Absolute percentage deviation from high-precision data
for various *ab initio* approximations

	$\langle 6p_{1/2} \| D \| 6s_{1/2} \rangle$	$A(6s_{1/2})$	$A(6p_{1/2})$
Singles-Doubles[a]	0.4%	5%	7%
SD + pert Sv[Tv][a]	1.2%	0.9%	0.8%
SD+Triples[IV][b]	0.6%	0.8%	0.1%
CCSDvT (May 2006)[c]	0.1-0.2%	0.1%	0.7%

a) Blundell, Johnson, Sapirstein (1991); Safronova, Johnson, Derevianko (1999)
b) Derevianko & Porsev Phys. Rev. A (2005)
c) Porsev & Derevianko (unpublished, May 2006)

radiative corrections needed

APV in chains of isotopes

Atomic structure out
Nuclear structure in

$$E_{PV} = k_{PV}\, Q_W^{\text{inferred}}$$

Ratio of PV amplitudes for two isotopes N and $N' = N + \Delta N$ of the same element (atomic-structure part is the same)

$$\text{R} = \frac{E_{\text{PV}}}{E'_{\text{PV}}} = \frac{Q_W}{Q'_W}\, f_{\text{nuclear}}\left(R_p, R'_p; R_n, R'_n\right)$$

- No atomic structure uncertainties *(Dzuba, Flambaum & Khriplovich (1986))*
- *Fortson et al., (1990)* – enhanced sensitivity to uncertainties in neutron radii R_n
- **Present uncertainties in *nuclear structure calculations* preclude extraction of new physics.**

Reanalysis using empirical data from anti-protonic experiments

$$\Delta R_{np} = \left(-0.04 \pm 0.03\right) + \left(1.01 \pm 0.15\right)\frac{N-Z}{A}\ \text{fm}$$

From Trzcinska *et al.*, PRL **87** 082501 (2001)

$$Q = Q_{SM} + \Delta Q_{new} = Q_{SM} + Zh_p^{new} + Nh_n^{new}$$

Constrains on "new direct physics'' (the tighter the better)

uncertainties in the skin

$$\delta F = \frac{\delta h_p^{\text{new}}}{h_n^{\text{SM}}} \approx \frac{N}{\Delta N}\left(\frac{N}{Z}\right)\left[\frac{\delta \text{R}}{\text{R}} + \frac{3}{7}\left(\alpha Z\right)^2 \frac{\delta\left(R_n - R_n'\right)}{R_p}\right]$$

exp error

Single-isotope vs isotopic chains

$$\Delta Q_{new} = Z h_p^{new} + N h_n^{new}$$

Chains are sensitive to proton couplings
Single isotope -- to neutron couplings

$$h_p^{new} = 2h_u + h_d \qquad h_n^{new} = 2h_d + h_u$$

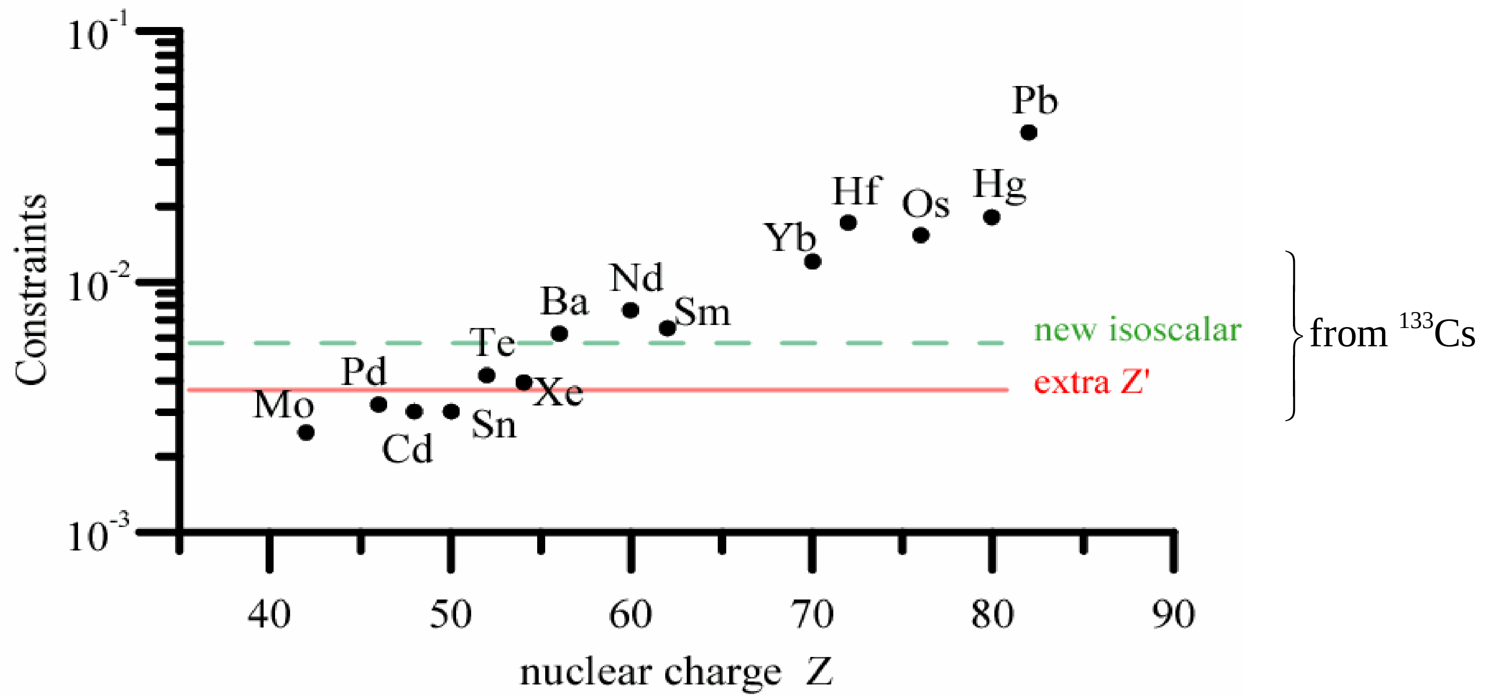
Ramsey-Musolf (1999)

$$h_u = \lambda h_d$$

Comparison is model-dependent

λ=1 pure isoscalar coupling
λ=0 extra neutral-gauge Z boson (E_6 models)

APV in chains of isotopes



- Atoms with Z < 50 may be of immediate interest at present
- Since $E_{PNC} \propto Z^3$, "accidental" degeneracy scenario is required

Derevianko and Porsev *Phys. Rev. A* ***65*** *052115 (2002)*

Summary APV

- ⇨ Single-isotope measurements
  - ⇨ Theory of “smaller corrections” is settled
  - ⇨ Agreement with the Standard Model @ 1 sigma
  - ⇨ Challenges for the next accuracy level:
    - ⇨ Correlations
    - ⇨ Improved measurements (Cs, Ba^+, Fr)
- ⇨ Isotopic-chains:
  - ⇨ Uncertainties in the neutron skin need to be reduced
  - ⇨ Z<50 could provide “new physics” bounds competitive to Cs