INSTRUCTIONS: Please try and do these problems without looking at the book. If you need help, solutions are provided at the end of this document. Your test questions will differ from this practice questions. **You may need the binomial tables for the exam-make sure you have copies with you!**

ENJOY!

1. True or false: Circle the correct answer in ink.
   
   T or F. If events A and B are independent, then events $A^c$ and $B^c$ (complements of A and B) are also independent.
   
   T or F. If P(A)=0.3 and P(B)=0.6, and P(A and B)=0.18, then events A and B are independent.
   
   T or F. If P(A) =0.5 and P(B)=0.1 and P(A or B)=0.4, then A and B must be mutually exclusive.
   
   T or F. Let X be a continuous random variable, with p.d.f. f(x). Then, for any two numbers $a$ and $b$ with $a < b$ we have $P(a<X<b)=\int_{a}^{b}f(x)dx$
   
   T or F. Cumulative distribution function is always a strictly increasing function.

2. Suppose you missed an exam in math 352. The professor said that she can assign you a grade which was a measure of center for the class grades. The distribution of grades is skewed to the right. Which would you chose: the mean or the median? Of course you want the larger one!

   (a) mean (b) median

3. The following figures represent maximal daily temperatures (in degrees Celsius) recorded on five December days in Chicago: -2, 2, -2, 0, 2. Please find the:

   (I) mean temperature (in degrees Celsius);

   (a) -1 (b) 0 (c) 1 (d) 2 (e) other than given in (a) - (d)

   (II) median temperature (in degrees Celsius);

   (a) -2 (b) 0 (c) 0.5 (d) 1 (e) other than given in (a) - (d)

   (III) standard deviation of temperatures (in degrees Celsius);

   (a) -1 (b) 1 (c) 1.789 (d) 2 (e) other than given in (a) - (d)

4. Suppose the mean annual salary of 10 employees at MyCo is $40,000 and their median salary is $37,000. If the lowest paid employee is the only one to suffer a pay-cut of $1,000 next year, what will be the new

   (I) mean annual salary of this group?

   (a) $40,000 (b) $39,900 (c) $39,800 (d) $37,000 (e) other than given in (a) - (d)

   (II) median annual salary of this group?

   (a) $40,000 (b) $39,900 (c) $38,900 (d) $37,000 (e) other than given in (a) - (d)
5. The mean and standard deviation of the daily high temperatures (in degrees Fahrenheit) in Reno on eight consecutive days are \( \bar{x} = 45 \) and \( s = 6.3 \). What are the corresponding mean and standard deviation of the temperatures in degrees Celsius? Note: To convert a temperature in degrees Fahrenheit to degrees Celsius use: \( C = \left(\frac{5}{9}\right)F - 32 \).

6. Consider the following observations on shear strength of a joint bonded in a particular manner:

\[
30.0 \ 4.4 \ 33.1 \ 66.7 \ 81.5 \ 22.2 \ 40.4 \ 16.4 \ 73.7 \ 36.6 \ 109.9
\]

a. Determine the value of the sample mean.
b. Determine the value of the sample median.

7. A large company offers its employees two different health insurance plans and two different dental insurance plans. Plan 1 of each type is relatively inexpensive, but restricts the choice of providers, whereas plan 2 is more expensive but more flexible. The accompanying table gives the percentages of employees who have chosen the various plans:

<table>
<thead>
<tr>
<th>Dental Plan</th>
<th>Health Plan 1</th>
<th>Health Plan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health Plan</td>
<td>27%</td>
<td>14%</td>
</tr>
<tr>
<td>1</td>
<td>24%</td>
<td>35%</td>
</tr>
</tbody>
</table>

Suppose that an employee is randomly selected and both the health plan and dental plan chosen by the selected employee are determined.

a. What are the events in the sample space?
b. What is the probability that the selected employee has chosen the more restrictive plan of each type?
c. What is the probability that the employee has chosen the more flexible dental plan?

8. Student Engineers Council at an Indiana college has one student representative from each of the five engineering majors (civil, electrical, industrial, materials, and mechanical). In how many ways can

a. Both a council president and a vice president be selected?
b. A president, a vice president, and a secretary be selected?

9. At a certain gas station, 40% of the customers use regular unleaded gas \( (A_1) \), 35% use extra unleaded gas \( (A_2) \), and 25% use premium unleaded gas \( (A_3) \). Of those customers using regular gas, only 30% fill their tanks (event \( B \)). Of those customers using extra gas, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

a. What is the probability that the next customer will request extra unleaded gas and fill the tank?
b. What is the probability that the next customer fills the tank?
c. If the next customer fills the tank, what is the probability that regular gas is requested? Extra gas? Premium gas?

10. A mail-order computer business has five telephone lines. Let \( X \) denote the number of lines in use at a specified time. Suppose the pmf of \( X \) is as given in the accompanying table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>.10</td>
<td>.15</td>
<td>.20</td>
<td>.25</td>
<td>.22</td>
<td>.08</td>
</tr>
</tbody>
</table>
Calculate the probability of each of the following events.

a. {at most 3 lines are in use}
b. {fewer than 3 lines are in use}
c. {at least 3 lines are in use}
d. {between 2 and 5 lines, inclusive, are in use}
e. {at least 4 lines are not in use}

11. An insurance company offers its policyholders a number of different payment options. For a randomly selected policyholder, let $X =$ the number of months between successive payments. The cdf of $X$ is as follows:

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \leq x < 3 \\ .40 & 3 \leq x < 4 \\ .45 & 4 \leq x < 6 \\ .60 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

a. What is the pmf of $X$?
b. Using just the cdf, compute $P(3 \leq X \leq 6)$ and $P(X \geq 4)$.
c. Using just the pmf, compute $P(X>6)$.

12. An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let $X =$ the amount of storage space purchased by the next customer to buy a freezer. Suppose that $X$ has pmf

<table>
<thead>
<tr>
<th>$x$</th>
<th>13.5</th>
<th>15.9</th>
<th>19.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>.2</td>
<td>.4</td>
<td>.4</td>
</tr>
</tbody>
</table>

a. Compute $E(X)$, $E(x^2)$, and $V(X)$.
b. If the price of a freezer having capacity $X$ cubic feet is $25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?
c. What is the variance of the price $25X - 8.5$ paid by the next customer?

13. Suppose that only 25% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions,

a. At most 6 will come to a complete stop?
b. Exactly 6 will come to a complete stop?
c. How many of the next 20 drivers do you expect to come to a complete stop?
a. 

14. A college professor always finishes his lectures within 2 minutes after the bell rings to end the lecture. Let $X =$ the time that elapses between the bell and the end of the lecture and suppose the pdf of $X$ is

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$
a. Find the value of $k$. [Hint: Total area under the graph of $f(x)$ is 1.]

b. What is the probability that the lecture ends within 1 minute of the bell ringing?

c. What is the probability that the lecture continues beyond the bell for between 60 and 90 seconds?

d. What is the probability that the lecture continues for at least 90 seconds beyond the bell?

15. The cdf of checkout duration $X$ for a book on a 2-hour reserve at a college library is given by:

$$
F(x) = \begin{cases} 
0 & x < 0 \\
\frac{x^2}{4} & 0 \leq x < 2 \\
1 & 2 \leq x 
\end{cases}
$$

Use this cdf to compute the following:

a. $P(X \leq 1)$

b. $P(.5 \leq X \leq 1)$

c. $P(X > .5)$

d. The median checkout duration $\tilde{\mu}$ [Hint: solve $.5 = F(\tilde{\mu})$]

e. Find $F'(x)$ to obtain the density function $f(x)$

16. Let $X$ denote the amount of space occupied by an article placed in a 1–ft³ packing container. The pdf of $X$ is

$$
f(x) = \begin{cases} 
90x^4(1-x) & 0 < x < 1 \\
0 & \text{otherwise}
\end{cases}
$$

a. Obtain the cdf of $X$.

b. What is $P(X \leq .5)$ [i.e., $F(.5)$]?

c. What is $P(.25 < X \leq .5)$?

d. Compute $E(X)$ and $\sigma_x$.

e. What is the probability that $X$ is within 1 standard deviation of its mean value?

ANSWERS

Problem 1. T, T, F, T, F

Problem 2. mean (a)

Problem 3. mean=0, median=0, st.dev=2.

Problem 4.

sum of old salaries=10*$40,000=$400,000
sum of new salaries= $400,000-$1,000=$399,000

(I) new mean salary=$39,900
(II) median will not change, new median=$37,000.
Problem 5.

\[
\text{mean temp C} = (5/9) \times (\text{mean temp F}) - 32 = (5/9) \times 45 - 32 = -7^\circ \text{C}
\]

\[
\text{st.dev. C} = (5/9) \times (\text{st.dev temp F}) = (5/9) \times 6.3 = 3.5^\circ \text{C}
\]

Problem 6.

a. The sum of the \( n = 11 \) data points is 514.90, so \( \bar{x} = 514.90/11 = 46.81 \).

b. The sample size (\( n = 11 \)) is odd, so there will be a middle value. Sorting from smallest to largest: 4.4 16.4 22.2 30.0 33.1 36.6 40.4 66.7 73.7 81.5 109.9. The sixth value, 36.6 is the middle, or median, value.

Problem 7.

Let \( H_1 \) and \( H_2 \) represent the two health plans. Let \( D_1 \) and \( D_2 \) represent the two dental plans.

a. The events in the sample space are pairs \{health plan, dental plan\}: \{\( H_1, D_1 \), \( H_1, D_2 \), \( H_2, D_1 \), \( H_2, D_2 \)\}.

b. \( P(H_1, D_1) = .27 \)

c. \( P(D_2) = P(H_1, D_2), \{H_2, D_2\} = .14 + .35 = .49 \)

Problem 8.

a. \( (5)(4) = 20 \) (5 choices for president, 4 remain for vice president)

b. \( (5)(4)(3) = 60 \)

Problem 9

\[
P(A_1) = .40, \ P(A_2) = .35, \ P(A_3) = .25
\]

\[P(B|A_1)=0.3, \ P(B|A_2)=0.6, \ \text{and} \ P(B|A_3)=0.5.\]

Therefore,

\[
P(A_1 \cap B) = P(A_1) \cdot P(B|A_1) = (.40)(.30) = .12
\]

\[
P(A_2 \cap B) = P(A_2) \cdot P(B|A_2) = (.35)(.60) = .21
\]

\[
P(A_3 \cap B) = P(A_3) \cdot P(B|A_3) = (.25)(.50) = .125
\]

a. \( P(A_3 \cap B) = .21 \)

b. \( P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .12 + .21 + .125 = .455 \)

\[
P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{.21}{.455} = .462
\]

\[
P(A_3|B) = \frac{P(A_3 \cap B)}{P(B)} = \frac{.125}{.455} = .275
\]

Problem 10.

a. \( P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = .70 \)

b. \( P(X < 3) = P(X \leq 2) = p(0) + p(1) + p(2) = .45 \)

c. \( P(X \geq 3) = p(3) + p(4) + p(5) = .55 \)

d. \( P(2 \leq X \leq 5) = p(2) + p(3) + p(4) + p(5) = .70 \)

e. \( P(X \leq 1) = .45 \)
Problem 1. a. Possible values are those values at which $F(x)$ jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>.30</td>
<td>.10</td>
<td>.05</td>
<td>.15</td>
<td>.40</td>
</tr>
</tbody>
</table>

b. $P(3 \leq X \leq 6) = F(6) - F(2) = 0.6 - 0.3 = 0.3$.

P($X \geq 4$) = 1 − $P(X \leq 3)$ = 1 − $F(3) = 1 - 0.4 = 0.6$.

c. $P(X > 6) = P(X = 12) = 0.4$.

Problem 2. a. $E(X) = (13.5)(.2) + (15.9)(.4) + (19.1)(.4) = 16.70$

$V(X) = (13.5)^2(.2) + (15.9)^2(.4) + (19.1)^2(.4) = 283.498$

b. $E(X - 8.5) = E(X) - 8.5 = (25)(16.70) - 8.5 = 409$

c. $V(X - 8.5) = V(X) = (25)^2(283.498) = 2880$

Problem 3. Let $S$ = driver comes to a complete stop, so $p = .25$, $n = 20$; $X$ is Binomial.(20, 0.25).

a. $P(X \leq 6) = \sum_{k=0}^{6}(20\choose k)0.25^k0.75^{20-k} = 0.786$ (MINITAB = 0.786)

b. $P(X = 6) = (20\choose 6)0.25^60.75^{14} 

c. $EX = 20(0.25) = 5$. We expect 5 of the next 20 to stop.

d. $V(X) = (20)(0.25)(0.75) = 3.75$

Problem 4. a. $1 = \int_{-\infty}^{\infty} f(x)dx = \int_{0}^{1} kx^2dx = k(x^3/3)|_0^1 = k(8/3) \Rightarrow k = 3/8 = .375$

b. $P(0 \leq X \leq 1) = \int_{0}^{1} .375x^2dx = .125 x^3|_0^1 = .125$

c. $P(1 \leq X \leq 1.5) = \int_{1}^{1.5} .375x^2dx = .125 x^3|_1^{1.5} = .2969$

d. $P(X \geq 1.5) = 1 - \int_{0}^{1.5} .375x^2dx = 1 - .125 x^3|_0^{1.5} = .5781$

Problem 5. a. $P(X \leq 1) = F(1) = \frac{1}{4} = .25$

b. $P(.5 \leq X \leq 1) = F(1) - F(.5) = \frac{3}{16} = .1875$

c. $P(X > .5) = 1 - P(X \leq .5) = 1 - F(.5) = \frac{15}{16} = .9375$

d. $.5 = F(\mu) = \frac{\mu^2}{4} \Rightarrow \mu^2 = 2 \Rightarrow \mu = \sqrt{2} = 1.414$

e. $f(x) = F'(x) = \frac{x}{2}$ for $0 \leq x < 2$, and = 0 otherwise

Problem 6. a. $F(x) = \int_{0}^{x} f(y)dy = \int_{0}^{90} 90y^8(1-y)dy = 90\int_{0}^{9}(y^8 - y^9)dy$

$= 90\left[\frac{1}{9} y^9 - \frac{1}{10} y^{10}\right]|_0^9 = 10x^9 - 9x^{10}$

Therefore,

$$F(x) = \begin{cases} 
0 & \text{for } x \leq 0 \\
10x^9 - 9x^{10} & \text{for } 0 < x < 1 \\
1 & \text{for } x \geq 1 
\end{cases}$$

b. $F(.5) = 10(.5)^9 - 9(.5)^{10} \approx .0107$
c.  
\[ P(.25 \leq X \leq .5) = F(.5) - F(.25) \approx .0107 - [10(.25)^9 - 9(.25)^{10}] \approx .0107 - .0000 \approx .0107 \]

d.  
\[ E(X) = \int_{-\infty}^{\infty} x \cdot f(x) \, dx = \int_{0}^{1} x \cdot 90x^8(1-x) \, dx = 90\int_{0}^{1} x^9(1-x) \, dx \]
\[ = 9x^{10} - \frac{90}{11}x^{11}\Big|_0^1 = \frac{9}{11} \approx .8182 \]
\[ E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x) \, dx = \int_{0}^{1} x^2 \cdot 90x^8(1-x) \, dx = 90\int_{0}^{1} x^{10}(1-x) \, dx \]
\[ = \frac{90}{11}x^{11} - \frac{90}{12}x^{12}\Big|_0^1 \approx .6818 \]
\[ V(X) \approx .6818 - (.8182)^2 = .0124, \quad \sigma = .11134. \]

e.  
\[ \mu \pm \sigma = (.7068, .9295). \quad \text{Thus,} \quad P(\mu - \sigma \leq X \leq \mu + \sigma) = F(.9295) - F(.7068) = .8465 - .1602 = .6863 \]