1. True or false: Circle the correct answer.

T or F. The Type I error in testing hypotheses: Ho: μ=2 vs. H1: μ≠ 2 is the probability that we conclude μ≠ 2 given that in reality μ=2.

T or F. If a null hypothesis is rejected at the 0.05 significance level, it has to be also rejected at the 0.01 significance level.

2. Let p denote the proportion of all potential subscribers who favor cable company A over cable company B. Consider testing Ho: p ≤ 0.5 versus H1: p > 0.5 based on a random sample of 9 individuals. Suppose that the null hypothesis is rejected if X ≥ 8, where X is the number of individuals in the sample who favor company A.

(a) Describe what Type I and Type II errors are in the context of this problem.

(b) What is the probability distribution of the test statistic X when H0 is true and p=0.5? Use it to derive the probability of Type I error.

(c) What would you conclude if 5 of the 9 individuals in the sample favored company B?

3. Let μ denote the true mean tread life of a certain type of tire. Consider a level α = .05 test of Ho: μ ≤ 20,000 versus H1: μ > 20,000 based on a sample of size n=16 from a normal population with σ=1500. Carry out the test if \( \bar{X} = 20,960 \).

4. In their annual report, the State Department of Education stated that 62% of academic employees in the State were permanent faculty and the remaining 38% were temporary. A professor in Bumbleton State University (BSU) claims that BSU has a lower proportion of permanent faculty than the overall State proportion of 62%. A random sample of 400 academic employees at BSU showed 215 of them to have permanent positions. Test the professor’s claim at 0.01 significance level.

5. The people representative claims that the true mean medical expenses during a year (for a family) are greater than $750. In a survey in which 100 randomly chosen middle-class families were interviewed, it was found that their mean medical expenses during a year were $770 with standard deviation of $120. Is the rep’s claim justified? Test appropriate hypothesis using a significance level α = 0.025.

6. A sample of 12 radon detectors of a certain type was selected, and each was exposed to 100 pCi/L of radon. The resulting readings were as follows:

104.3  89.6  89.9  95.6  95.2  90.0  98.8  103.7  98.3  106.4  102.0  91.1

Assume that radon readings come from a normal distribution. Does this data suggest that the population mean reading under these conditions differs from 100? State and test the appropriate hypotheses using significance level α =.05
7. State DMV records indicate that of all vehicles undergoing emissions testing during the previous year, 70% passed on the first try. A random sample of 200 cars tested in a particular county during the current year yields 160 that passed on the initial test. Does this suggest that the true proportion for this county during the current year differs from the previous statewide proportion? Test the relevant hypotheses using $\alpha = 0.05$.

8. A breeder of rabbits claims that he can breed rabbits yielding a mean weight of greater than 58 ounces. A random sample of 16 rabbits had a mean weight of 59.2 ounces and standard deviation of weights $s = 3$ ounces. Assume normal distribution of rabbits’ weights. Is the breeder’s claim justified? Use 5% level of significance.

9. In a study of the effectiveness of a pesticide on an oats leaf beetle, researchers measured the number of beetle larvae per stem. One plot was treated with a pesticide, another was not (control). 13 stems were inspected on the plot treated with a pesticide and yielded mean number of larvae per stem of 1.36 with a standard deviation 0.52. 14 stems were inspected from the control plot and yielded an average of 3.47 larvae per stem with a standard deviation 1.21. Is there significant evidence that the pesticide is effective? Use significance level of 1%. Assume distribution of larvae per stem to be approximately normal.

Let $\mu_x =$ mean larvae per stem with pesticide, and $\mu_y =$ mean larvae per stem with control. Finally, let the alternative hypothesis be: $H_A: \mu_x < \mu_y$.

10. Twenty men participated in a weight loss program. Their weight was measured before and after the program. The differences of weight before minus weight after the program were computed for all men. The mean difference weight was 2.5 kg with a standard deviation of differences of 0.5 kg. At the 1% level of significance, was the weight loss program effective? Assume normal distribution of weights.

11. A medical study was looking into the question if using a drug to reduce blood cholesterol level will decrease the risk of a heart attack. Middle age men were randomly assigned to two groups. One group of 2051 men took a drug that reduces cholesterol level, and the second group (control) of 2030 men took a placebo. During the next five years, 56 men from the treatment group and 84 men from the control group had heart attacks. Was the drug treatment effective? Use 5% significance level.

Let $P_t =$ proportion of suffering heart attacks in treatment group

$P_c =$ proportion of suffering heart attacks in control group

Further, let the alternative hypothesis be $H_A: P_t < P_c$.

12. To measure the difference between average weights of Labrador and Golden Retrievers, 20 dogs of each kind were randomly selected. The average weight of the Labradors was 70 pounds, while the average weight of the Goldens was 75 pounds. The population standard deviation ($\sigma$) of weights for the Labradors is 1.5 pounds and for the Goldens is 2.3 pounds. Are the Golden Retrievers significantly heavier than the Labrador Retrievers? Use significance level of 0.01. Let $\mu_x =$ mean weight of a Labrador; $\mu_y =$ mean weight of a Golden Retriever. Further, let the alternative hypothesis be $H_A: \mu_x < \mu_y$. Assume weights of Labs are approximately normal.

13. Human Resource departments often notice that there seems to be a relationship between the number of hours the employees are expected to work per week and the number of sick days they take. To investigate this relationship, a management consultant for company X interviewed random sample of 15 employees. The consultant asked the employees about the number of hours they are expected to work (X) and how often they have been sick enough to miss work last year (Y). The results and summary statistics are below.
\[
\begin{array}{cccccccccccc}
X & 50 & 55 & 45 & 37 & 40 & 30 & 60 & 55 & 52 & 50 & 45 & 40 & 35 & 55 & 50 \\
Y & 4 & 5 & 5 & 2 & 4 & 0 & 6 & 3 & 2 & 5 & 2 & 2 & 3 & 2 & 1 \\
\end{array}
\]

\[\Sigma X=699 \quad \Sigma Y=46 \quad \Sigma X^2=33,623 \quad \Sigma Y^2=182 \quad \Sigma XY=2,248\]

a) Find the correlation coefficient between the number of hours worked (X) and the number of days missed (y).

a. 0.504  
   b. 0.043  
   c. 0.742  
   d. 0.052

b) Conduct a test to determine if the correlation between the number of hours worked (X) and the number of days missed (y) is significantly different from zero. Use significance level of 1%.
Problem 1. T, F.

Problem 2.

a. Type I error = reject Ho when it is true = decide that over 50% of potential subscribers prefer Company A, when in fact at most 50% of subscribers prefer A.

Type II error = accept Ho when it is false = decide that at most 50% of potential subscribers prefer Company A, when in fact more than 50% of subscribers prefer A.

b. The test statistic in this problem is \( X = \) number of individuals in the sample of 9 who prefer A. When Ho is true and \( p=0.5 \), then the distribution of \( X \) is Binomial(9, 0.5).

The decision rule given in the problem reject Ho when \( X \geq 8 \). Thus,

\[
P(\text{Type I error}) = P(X \geq 8 \mid X \text{ is Binomial}(9, 0.5)) = 1 - P(X \leq 7) \]

\[
= \text{MINITAB} = 1 - 0.9805 = 0.0195.
\]

**Cumulative Distribution Function**

<table>
<thead>
<tr>
<th>x</th>
<th>P(X &lt;= x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.980469</td>
</tr>
</tbody>
</table>

c. If \( X=5 \), then using the decision rule described in the problem, we would not reject Ho, and conclude that the proportion of potential subscribers who prefer Company A does not exceed 0.5.

Problem 3. \( \mu \)=true mean thread life of a tire, \( n=16, \sigma=1500, X = 20,960 \), normal population of lifetimes. Test hypotheses:

\[ \text{Ho: } \mu \leq 20,000 \text{ versus } \text{H1: } \mu > 20,000 \]

Test statistic

\[
z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{20,960 - 20,000}{1500 / \sqrt{16}} = 2.56 \text{ (I used MINITAB)}
\]

Critical number: \( z(0.05) = 1.645 \), Decision rule: reject Ho when the test statistic > 1.645

**One-Sample Z**

Test of mu = 20000 vs not = 20000
The assumed standard deviation = 1500

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>95% CI</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>20960</td>
<td>375 (20225, 21695)</td>
<td><strong>2.56</strong></td>
<td>0.010</td>
<td></td>
</tr>
</tbody>
</table>
Problem 4. Professor’s claim: Test the professor’s claim at 0.01 significance level.

Step 1. $H_0$: $p = 0.62$  \quad $H_A$: $p < 0.62$ (professor’s claim)

Step 2: value of the test statistics $= z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = -3.39935$

Step 3: Critical value(s) = -2.326

Step 4: Decision: Since the test statistic $z=-3.399 < -2.326 =$ critical number, then we reject $H_0$.

Step 5: Is the professor’s claim justified? Yes

Problem 5.

Let $\mu =$ true mean family medical expenses (in $), n=100, \bar{X} =770, and s=120. Claim: the mean medical expenses are greater than 750. Significance level = 0.025.

Step 1. $H_0$: $\mu \leq 750$  \quad $H_A$: $\mu > 750$ (claim of the people rep)

Step 2: value of the test statistics $= t = \frac{\bar{X} - \mu_0}{s} = 1.67$

Step 3: Critical value comes from t distribution with 99 df. Since we do not have 99 df in the table, I used 120 df and thus $t(0.025) = 1.98$.

Step 4: Decision: Since the test statistic $z= 1.67 < 1.98 =$ critical number, then we do not reject $H_0$.

Step 5: Is the people rep’s claim justified? No

Problem 6. Radon detectors

Let $\mu =$ true mean reading for exposure to 100 pCi/L. $n=12, \bar{X} = 97.07, and s=6.11. Claim: the mean reading is different from 100. Significance level = 0.05.

Step 1. $H_0$: $\mu =100$  \quad $H_A$: $\mu \neq 100$ (mean reading differs from 100)

Step 2: value of the test statistics $= t = \frac{\bar{X} - \mu_0}{s} = -1.66$

Step 3: Critical values come from t distribution with 11 df, $t(0.025) = 2.201$, critical numbers: $\pm 2.201$.

Step 4: Decision: Since the test statistic $-2.201 < t=-1.66 < 2.201$, then we do not reject $H_0$.

Step 5: Is the mean reading different from 100? No, the data do not indicate that mean reading differs significantly from 100.

One-Sample T: reading
Test of $\mu = 100$ vs not $= 100$

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% CI</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>reading</td>
<td>12</td>
<td>97.07</td>
<td>6.11</td>
<td>1.76</td>
<td>(93.19, 100.96)</td>
<td>-1.66</td>
<td>0.125</td>
</tr>
</tbody>
</table>

**Problem 7.** DMV problem

Parameter of interest: $p =$ true proportion of cars in this particular county passing emissions testing on the first try. The sample proportion is $\hat{p} = 160 / 200 = 0.80$, significance level is 0.05.

$H_0 : p = 0.70$, and $H_a : p \neq 0.7$

The test statistic value is $z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = 3.086$

Critical numbers: $z(0.025)=1.96$, so critical numbers: $\pm 1.96$. Since the test statistic $= 3.086 > 1.96$, we reject $H_0$.

The data indicates that the proportion of cars passing the first time on emission testing or this county differs from the proportion of cars passing statewide.

**Test and CI for One Proportion**

Test of $p = 0.7$ vs $p \neq 0.7$

<table>
<thead>
<tr>
<th>Sample X</th>
<th>N</th>
<th>Sample p</th>
<th>95% CI</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160</td>
<td>0.800000</td>
<td>(0.737774, 0.853106)</td>
<td>0.002</td>
</tr>
</tbody>
</table>

**Problem 8. Rabbit problem**;

Let $\mu =$mean weight of rabbit

a. Step 1. $H_0: \mu \leq 58$ $H_A: \mu > 58$ (breeder’s claim)

Step 2:
Test statistic: $t_0 = (\bar{x} - 58)/(s/\sqrt{n}) = (59.2 - 58)/(3/\sqrt{4}) = 1.6$

Step 3: Critical value(s) = 1.75

Critical value $= t_{n-1}, \alpha = t_{15,.05} = 1.75$

Step 4: Decision: Do not reject $H_0$.

Step 5: Is the breeder’s claim justified? **No**, the breeder’s claim for $\mu > 58$ is NOT justified!

**Problem 9. Larvae problem.** Let $\mu_x =$ mean larvae per stem with pesticide; $\mu_y =$ mean larvae per stem with control
H₀: \( \mu_x = \mu_y \) \\
Hₐ: \( \mu_x < \mu_y \)

Given \( m = 13, \bar{x} = 1.36, S_x = .52 \); \( n = 14, \bar{y} = 3.47, S_y = 1.21 \), \( \alpha = 0.01 \)

1. value of the test statistics =

\[
t = \frac{1.36 - 3.47}{\sqrt{\frac{0.52^2}{13} + \frac{1.21^2}{14}}} = \frac{-2.11}{0.354} = -5.96
\]

2. The distribution of the test statistic under Ho is t with 17 df.
4. Critical number: -2.567
5: Decision: Reject Ho
6: Is there significant evidence that the pesticide is effective? YES
Since mean larvae per stem with pesticide is less than that with control, we conclude that the pesticide is effective.

**Problem 10. Weight loss problem.** Step 0. Let \( \mu_x = \) mean weight before program; \( \mu_y = \) mean weight after program

\( \mu_d = \) mean difference in weight after program = \( \mu_x - \mu_y \)

H₀: \( \mu_d = 0 \) \\
Hₐ: \( \mu_d > 0 \)

1. value of the test statistics =

\[
t = \frac{\bar{d}}{S_d / \sqrt{n}} = \frac{2.5}{.5/\sqrt{20}} = 22.36
\]

2. The distribution of the test statistic under Ho is t with 19 df.
3: p-value ≈ 0
4. Critical number: 2.539
5: Decision: Reject H₀.
6: Is there significant evidence that the weight loss program was effective? YES
Since average weight after the program is less than the one before, we say that the weight loss program was effective.

**Problem 11. Medication problem.** Let \( P_t = \) proportion of suffering heart attacks in treatment group

\( P_c = \) proportion of suffering heart attacks in control group

H₀: \( P_t = P_c \) \\
Hₐ: \( P_t < P_c \)

1. value of the test statistics =

\[
Z = \frac{56}{2051} - \frac{84}{2030} = -2.4701, \quad \hat{p} = \frac{(56+84)}{(2051+2030)} = 0.03431
\]

\[
S_{\hat{p}}^2 = \hat{p}(1-\hat{p})\left[\frac{1}{2051} + \frac{1}{2030}\right] = 0.0000325 \Rightarrow S_{\hat{p}} = 0.005698
\]
2. The distribution of the test statistic under Ho is standard normal
3. p-value = 0.0068
4. Critical number: - 1.645
5. Decision: Reject Ho
6. Was the drug treatment effective? Yes
Since proportion of suffering heart attacks are less in treatment group than that in control group, we say that drug treatment is effective.

Problem 12. Dog problem. Let $\mu_x =$ mean weight of a Labrador; $\mu_y =$ mean weight of a Golden Retriever

$H_0: \mu_x = \mu_y$
$H_A: \mu_x < \mu_y.$

Given $m = 20, \bar{x} = 70, \sigma_x = 1.5$; $n = 20, \bar{y} = 75, \sigma_y = 2.3$, $\alpha = 0.01$

1. value of the test statistics =
Since the population standard deviations are known,

$$Z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{m} + \frac{\sigma_y^2}{n}}} = \frac{70 - 75}{\sqrt{\frac{1.5^2}{20} + \frac{2.3^2}{20}}} = \frac{-5}{.614} = -8.14$$

2. The distribution of the test statistic under Ho is standard normal
3. p-value $\approx 0$
4. Critical number: - 2.326
5. Decision: Reject Ho

Step 6: Are the Golden Retrievers significantly heavier than the Labrador Retrievers? YES
Since the mean weight of Golden Retrievers is greater than that of Labradors, we say that Golden Retrievers are significantly heavier than the Labrador.

Problem 13. Correlation.

I used MINITAB to get the correlation coefficient and p-value for the test:

Correlations: hrs worked, days missed

Pearson correlation of hrs worked and days missed = 0.504
P-Value = 0.056

You can also do it using the formula:

$$n = 15$$

$$r_{xy} = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}} = \frac{15 \times 2248 - 699 \times 46}{\sqrt{(15 \times 33623 - 699^2)(15 \times 182 - 46^2)}} = .504$$

a) correlation coefficient is 0.504
b) Ho: rho = 0 Ha: rho not equal zero
test statistic \[ t = \frac{r}{\sqrt{1 - r^2}} \sqrt{n - 2} \]

substituting \( r=0.504 \), we get test stat \( t = 2.104 \). P-value = \( 2*P(t(13) > 2.104) = 2*(1 - 0.972296) = 0.055 \).

This is almost the same p-value that MINITAB reported. Since the significance level is 0.01, and p-value = 0.055 > 0.01, then we do not reject Ho, and conclude that the correlation between the number of hours worked (X) and the number of days missed (y) is not significantly different from zero.