LARGE SAMPLE TESTS FOR PROPORTIONS

Binomial experiment with n trials and unknown proportion of successes p.

GOAL: Test  \( H_0: p = p_0 \) (\( p \geq p_0 \) or \( p \leq p_0 \)) where \( p_0 \) is a specified, known value.

DATA: Observe \( x \) successes, get sample proportion of successes \( \hat{p} = \frac{x}{n} \).

Use the fact that statistic \( z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \) has appx N(0, 1) distribution under \( H_0 \) (when \( p = p_0 \)) for large n.

TEST FOR PROPORTION: PROCEDURE: Let significance level = \( \alpha \).

STEP 1. \( H_0: p = p_0 \) (\( \leq \) or \( \geq \)) vs \( H_a: p \neq p_0 \) or \( (H_a: p > p_0 \text{ or } H_a: p < p_0) \)

STEP 2. Compute the test statistic:
\[
z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}
\]

STEP 3. Find the critical number in the Z table.
Two sided alternative, then the critical value = \( z_{\alpha/2} \).
One sided alternative, then the critical values are \( z_\alpha \) or \( -z_\alpha \).

STEP 4. DECISION.
\begin{align*}
&\text{Ha: } p \neq p_0 \text{ Reject } H_0 \text{ if } |z| > z_{\alpha/2}; \\
&\text{Ha: } p > p_0 \text{ Reject } H_0 \text{ if } z > z_\alpha; \\
&\text{Ha: } p < p_0 \text{ Reject } H_0 \text{ if } z < -z_\alpha.
\end{align*}

STEP 5. Answer the question in the problem.

*****************************  P-value approach  **************************************

STEP 3. Compute an approximate p-value.
Two sided test p-value: Ha: \( p \neq p_0 \), approximate P-value: \( 2P(Z > |z|) \)
One sided tests p-values: Ha: \( p > p_0 \), approximate P-value: \( P(Z > z) \)
Ha: \( p < p_0 \), approximate P-value: \( P(Z < z) \)

STEP 4. DECISION: Reject \( H_0 \) if \( p \)-value < significance level \( \alpha \).

STEP 5. Answer the question in the problem.

Example: A survey of \( n = 880 \) randomly selected adult drivers showed that 56% of those respondents admitted to running red lights. Test the claim that the majority of all adult drivers admit to running red lights on significance level 0.01.

Example. When a coin is tossed 100 times, we get 60 Heads. Test if the coin is fair versus the alternative that it is loaded in favor of Heads, using significance level of 5%.

Example: When Gregory Mendel conducted his famous hybridization experiments with peas, one such experiment resulted in offspring consisting of 428 peas with green pods and 152 peas with yellow pods. According to Mendel’s theory, 1/4 of the offspring peas should have yellow pods. Use a 0.05 significance level with the \( P \)-value method to test the claim that the proportion of peas with yellow pods is equal to 1/4.
6.4 ONE SAMPLE t-TEST FOR THE MEAN OF THE NORMAL DISTRIBUTION

Let $X_1, \ldots, X_n$ sample from $N(\mu, \sigma)$, $\mu$ and $\sigma$ unknown, estimate $\sigma$ using $s$. Let significance level $= \alpha$.

**STEP 1.** $H_0: \mu = \mu_0$ \( (\leq \) or \( \geq \)) $H_a: \mu \neq \mu_0$ or \( (H_a: \mu > \mu_0 \) or \( H_a: \mu < \mu_0)\)

**STEP 2.** Compute the test statistic: 
$$ t = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} $$

**STEP 3.** Compute the critical number/value which depends on $H_a$.
Two sided alternative, then critical value $= t_{\alpha/2}(n-1)$.
One sided alternative, then critical value $= t_{\alpha}(n-1)$.

**STEP 4.** DECISION-critical/rejection regions, use $t$ distribution with $df=n-1$.
$H_a: \mu \neq \mu_0$ Reject $H_0$ if $|t| > t_{\alpha/2}(n-1)$;
$H_a: \mu > \mu_0$ Reject $H_0$ if $t > t_{\alpha}(n-1)$;
$H_a: \mu < \mu_0$ Reject $H_0$ if $t < -t_{\alpha}(n-1)$.

**STEP 5.** Answer the question in the problem.

*************** P-value approach ***************

**STEP 3.** Compute the $p$-value.
Two sided test $p$-value: $H_a: \mu \neq \mu_0$, $p$-value: $2P( t(n-1)> |t|)$

One sided tests $p$-values: $H_a: \mu > \mu_0$, $p$-value: $P( T_{n-1} > t)$
$H_a: \mu < \mu_0$, $p$-value: $P( T_{n-1} < t)$

**STEP 4.** DECISION: Reject $H_0$ if $p$-value < significance level $\alpha$.

**STEP 5.** Answer the question in the problem.

**Example:** Use $t$-table to find a range of values for the $P$-value corresponding to the given results.

a) In a left-tailed test, the sample size is 12, and the test statistic is $t = -2.007$.
b) In a two-tailed test, the sample size is 12, and the test statistic is $t = -3.456$.

**Example:** A sample of 36 women resulted in mean height of 64” and sample variance = 25. Are women, on average, shorter than 66”? Use 5% significance level. Assume heights follow normal distribution.

**Example:** Suppose a sample of size 16 from a normal distribution results in a mean of 10 and standard deviation of 3.2. Test $H_0: \mu = 8$ \( \) vs. \( H_a: \mu \neq 8 \) using $\alpha=0.05$.

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