Section 1.1

4. (a) False  
(b) True

5. (a) No.  
What is important is the population proportion of defectives; the sample proportion is only an approximation. The population proportion for the new process may in fact be greater or less than that of the old process.  
(b) No.  
The population proportion for the new process may be 0.12 or more, even though the sample proportion was only 0.11.  
(c) Finding 2 defective circuits in the sample.

Section 1.2

1. False

3. No. In the sample 1, 2, 4 the mean is 7/3, which does not appear at all.

5. The sample size can be any odd number.

6. Yes. For example, the list 1, 2, 12 has an average of 5 and a standard deviation of 6.08.

8. The mean increases by $50; the standard deviation is unchanged.

10. (a) Let $X_1, \ldots, X_{100}$ denote the 100 numbers of children.

$$\sum_{i=1}^{100} X_i = 27(0) + 22(1) + 30(2) + 12(3) + 7(4) + 2(5) = 156$$

$$\overline{X} = \frac{\sum_{i=1}^{100} X_i}{100} = \frac{156}{100} = 1.56$$

(b) The sample variance is

$$s^2 = \frac{1}{99} \left( \sum_{i=1}^{100} X_i^2 - 100 \overline{X}^2 \right)$$

$$= \frac{1}{99} [(27)^2 + (22)^2 + (30)^2 + (12)^2 + (7)^2 + (2)^2 - 100(1.56^2)]$$

$$= 1.7034$$

The standard deviation is $s = \sqrt{s^2} = 1.3052$.  
Alternatively, the sample variance can be computed as

$$s^2 = \frac{1}{99} \sum_{i=1}^{100} (X_i - \overline{X})^2$$

$$= \frac{1}{99} [27(0 - 1.56)^2 + 22(1 - 1.56)^2 + 30(2 - 1.56)^2 + 12(3 - 1.56)^2 + 7(4 - 1.56)^2 + 2(5 - 1.56)^2]$$

$$= 1.7034$$
(c) The sample median is the average of the 50th and 51st value when arranged in order. Both these values are equal to 2, so the median is 2.
(d) The first quartile is the average of the 25th and 26th value when arranged in order. Both these values are equal to 0, so the first quartile is 0.
(e) Of the 100 women, 30 + 12 + 7 + 2 = 51 had more than the mean of 1.56 children, so the proportion is 51/100 = 0.51.
(f) The quantity that is one standard deviation greater than the mean is 1.56 + 1.3052 = 2.8652. Of the 100 women, 12 + 7 + 2 = 21 had more than 2.8652 children, so the proportion is 21/100 = 0.21.
(g) The region within one standard deviation of the mean is 1.56±1.3052 = (0.2548, 2.8652). Of the 100 women, 22 + 30 = 52 are in this range, so the proportion is 52/100 = 0.52.

14. (a) We will work in units of $1000. Let $S_0$ be the sum of the original 10 numbers and let $S_1$ be the sum after the change. Then $S_0/10 = 70$, so $S_0 = 700$. Now $S_1 = S_0 - 100 + 1000 = 1600$, so the new mean is $S_1/10 = 160$.
(b) The median is unchanged at 55.
(c) Let $X_1, \ldots, X_{10}$ be the original 10 numbers. Let $T_0 = \sum_{i=1}^{10} X_i^2$. Then the variance is $(1/9)[T_0 - 10(70^2)] = 20^2 = 400$, so $T_0 = 52,600$. Let $T_1$ be the sum of the squares after the change. Then $T_1 = T_0 - 100^2 + 1000^2 = 1,042,600$. The new standard deviation is $\sqrt{(1/9)[T_1 - 10(160^2)]} = 295.63$.

16. (a) Seems certain to be an error.
(b) Could be correct.

Section 1.3

1.

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</table>
(b) Here is one histogram. Other choices for the endpoints are possible.

The boxplot shows one outlier.

6. (a) The histogram should be skewed to the right. Here is an example.
7. (a) The proportion is the sum of the relative frequencies (heights) of the rectangles above 130. This sum is approximately 
0.12+0.045+0.045+0.02+0.005+0.005 = 0.24. This is closest to 25%.
(b) The height of the rectangle over the interval 130–135 is greater than the sum of the heights of the rectangles over the interval 140–150. Therefore there are more women in the interval 130–135 mm.

12. The mean. The median, and the first and third quartiles are indicated directly on a boxplot, and the interquartile range can be computed as the difference between the first and third quartiles.

14. (a) False
(b) True
(c) False
(d) False

Section 2.1

1. \( P(\text{does not fail}) = 1 - P(\text{fails}) = 1 - 0.12 = 0.88 \)

3. (a) The outcomes are the 16 different strings of 4 true-false answers. These are \{TTTT, TTTF, TTFT, TFFF, TFFT, TFTF, TFFF, FTFT, FTTF, FFFF, FFTF, FFTF, FFFT, FFFF\}.
(b) There are 16 equally likely outcomes. The answers are all the same in two of them, TTTT and FFFF. Therefore the probability is 2/16 or 1/8.
(c) There are 16 equally likely outcomes. There are four of them, TFFF, FTFF, FFTF, and FFFT, for which exactly one answer is “True.” Therefore the probability is 4/16 or 1/4.
(d) There are 16 equally likely outcomes. There are five of them, TFFF, FTFF, FFTF, FFFF, and FFFF, for which at most one answer is “True.” Therefore the probability is 5/16.

7. (a) \( P(\text{living room or den}) = P(\text{living room}) + P(\text{den}) \)
   = 0.26 + 0.22
   = 0.48
(b) \( P(\text{not bedroom}) = 1 - P(\text{bedroom}) \)
\[= 1 - 0.37 \]
\[= 0.63 \]

9. (a) The events of having a major flaw and of having only minor flaws are mutually exclusive. Therefore \( P(\text{major flaw or minor flaw}) = P(\text{major flaw}) + P(\text{only minor flaws}) = 0.15 + 0.05 = 0.20. \)
(b) \( P(\text{no major flaw}) = 1 - P(\text{major flaw}) = 1 - 0.05 = 0.95. \)

12. (a) \( P(V \cap W) = P(V) + P(W) - P(V \cup W) \)
\[= 0.15 + 0.05 - 0.17 \]
\[= 0.03 \]
(b) \( P(Vc \cap Wc) = 1 - P(V \cup W) = 1 - 0.17 = 0.83. \)
(c) We need to find \( P(V \cap Wc). \) Now \( P(V) = P(V \cap W) + P(V \cap Wc) \) (this can be seen from a Venn diagram). We know that \( P(V) = 0.15, \) and from part (a) we know that \( P(V \cap W) = 0.03. \) Therefore \( P(V \cap Wc) = 0.12. \)

15. (a) Let \( R \) be the event that a student is proficient in reading, and let \( M \) be the event that a student is proficient in mathematics. We need to find \( P(Rc \cap M). \) Now \( P(M) = P(R \cap M) + P(Rc \cap M) \) (this can be seen from a Venn diagram). We know that \( P(M) = 0.78 \) and \( P(R \cap M) = 0.65. \) Therefore \( P(Rc \cap M) = 0.13. \)
(b) We need to find \( P(R \cap Mc). \) Now \( P(R) = P(R \cap M) + P(R \cap Mc) \) (this can be seen from a Venn diagram). We know that \( P(R) = 0.85 \) and \( P(R \cap M) = 0.65. \) Therefore \( P(R \cap Mc) = 0.20. \)
(c) First we compute \( P(R \cup M): \)
\[P(R \cup M) = P(R) + P(M) - P(R \cap M) = 0.85 + 0.78 - 0.65 = 0.98. \]
Now \( P(Rc \cap Mc) = 1 - P(R \cup M) = 1 - 0.98 = 0.02. \)

17. \( P(A \cap B) = P(A) + P(B) - P(AU B) \)
\[= 0.98 + 0.95 - 0.99 \]
\[= 0.94 \]

Section 2.2

2. (5)(2)(4) = 40

4. \( \binom{10}{5} = \frac{10!}{5!5!} = 252 \)

6. (8)(7)(6) = 336

Section 2.3

3. (a) 2/10
(b) Given that the first fuse is 10 amps, there are 9 fuses remaining of which 2 are 15 amps. Therefore \( P(\text{2nd is 15 amp}|\text{1st is 10 amp}) = 2/9. \)
(c) Given that the first fuse is 15 amps, there are 9 fuses remaining of which 1 is 15 amps. Therefore \( P(\text{2nd is 15 amp}|\text{1st is 15 amp}) = 1/9. \)

5. Given that a student is an engineering major, it is almost certain that the student took a calculus course. Therefore \( P(B|A) \) is close to 1. Given that a student took a calculus course, it is much less certain that the student is an engineering major, since many non-engineering majors take calculus. Therefore \( P(A|B) \) is much less than 1, so \( P(B|A) > P(A|B). \)
9. Let \( T \) denote the event that a person buys a hybrid vehicle, and let \( T \) denote the event that a person buys a hybrid truck. Then

\[
P(T | V) = \frac{P(T \cap V)}{P(V)} = \frac{P(T)}{P(V)} = \frac{0.05}{0.12} = 0.417
\]

13. Let \( T_1 \) denote the event that the first device is triggered, and let \( T_2 \) denote the event that the second device is triggered. Then \( P(T_1) = 0.9 \) and \( P(T_2) = 0.8 \).

(a) \( P(T_1 \cup T_2) = P(T_1) + P(T_2) - P(T_1 \cap T_2) \)
\[
= P(T_1) + P(T_2) - P(T_1)P(T_2)
\]
\[
= 0.9 + 0.8 - (0.9)(0.8) = 0.98
\]

(b) \( P(T_1^c \cap T_2^c) = P(T_1^c)P(T_2^c) = (1-0.9)(1-0.8) = 0.02 \)

(c) \( P(T_1 \cap T_2) = P(T_1)P(T_2) = (0.9)(0.8) = 0.72 \)

(d) \( P(T_1 \cap T_2^c) = P(T_1)P(T_2^c) = (0.9)(1-0.8) = 0.18 \)

15. (a) \( \frac{88}{88+12} = 0.88 \)

(b) \( \frac{88}{88 + 165 + 260} = 0.1715 \)

(c) \( \frac{88 + 165}{88 + 65 + 260} = 0.4932 \)

(d) \( \frac{88 + 165}{88 + 12 + 165 + 35} = 0.8433 \)

16. \( P(E_1) = \frac{88 + 165 + 260}{600} = \frac{513}{600} = 0.855 \). From Problem 15(a), \( P(E_2 | E_1) = 0.88 \). Since \( P(E_2 | E_1) \neq P(E_1) \), \( E_1 \) and \( E_2 \) are not independent.

24. (a) \( P(A) = \frac{300}{1000} = \frac{3}{10} \)

(b) Given that \( A \) occurs, there are 999 components remaining, of which 299 are defective. Therefore \( P(B | A) = \frac{299}{999} \).

(c) \( P(A \cap B) = P(A)P(B | A) = \left( \frac{3}{10} \right) \left( \frac{299}{999} \right) = \frac{299}{3330} \)
(d) Given that \(A^c\) occurs, there are 999 components remaining, of which 300 are defective. Therefore \(P(B|A^c) = 300/999\). Now \(P(A^c \cap B) = P(A^c)P(B|A^c) = (7/10)(300/999) = 70/333.

(e) \(P(B) = P(A \cap B) + P(A^c \cap B) = 299/3330 + 70/333 = 3/10\)

(f) \(P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{299/3330}{3/10} = \frac{299}{999}\)

(g) \(A\) and \(B\) are not independent, but they are very nearly independent. To see this note that \(P(B) = 0.3\), while \(P(B|A) = 0.2993\). So \(P(B)\) is very nearly equal to \(P(B|A)\), but not exactly equal. Alternatively, note that \(P(A \cap B) = 0.0898\), while \(P(A)P(B) = 0.09\). Therefore in most situations it would be reasonable to treat \(A\) and \(B\) as though they were independent.

32. Let \(F\) denote the event that a bottle has a flaw. Let \(F\) denote the event that a bottle fails inspection. We are given \(P(F) = 0.0002\), \(P(F|F) = 0.995\), and \(P(F^c|F^c) = 0.99\).

(a) \(P(F|F) = \frac{P(F|F)P(F)}{P(F|F)P(F) + P(F^c|F)P(F^c)} = \frac{P(F|F)P(F)}{P(F|F)P(F) + (1 - P(F|F^c))P(F^c)} = \frac{0.995(0.0002)}{(0.995)(0.0002) + (1 - 0.99)(0.9998)} = 0.01952\)

(b) i. Given that a bottle failed inspection, the probability that it had a flaw is only 0.01952.

(c) \(P(F^c|F^c) = \frac{P(F^c|F^c)P(F^c)}{P(F^c|F^c)P(F^c) + P(F^c|F)P(F)} = \frac{P(F^c|F^c)P(F^c)}{P(F^c|F^c)P(F^c) + (1 - P(F|F^c))P(F)} = \frac{0.9999999}{(0.9999999) + (1 - 0.9995)(0.0002)} = 0.9999999\)

(d) ii. Given that a bottle passes inspection, the probability that it has no flaw is 0.9999999.

(e) The small probability in part (a) indicates that some good bottles will be scrapped. This is not so serious. The important thing is that of the bottles that pass inspection, very few should have flaws. The large probability in part (c) indicates that this is the case.

35. \(P(C|D) = P(A \cap B) \cap (C \cup D)\). Now \(P(A \cap B) = P(A)P(B) = (1 - 0.05)(1 - 0.03) = 0.9215\), and \(P(C \cup D) = P(C) + P(D) - P(C \cap D) = (1 - 0.07) + (1 - 0.14) - (1 - 0.07)(1 - 0.14) = 0.9902\). Therefore

\[P((A \cap B) \cap (C \cup D)) = P(A \cap B)P(C \cup D) = (0.9215)(0.9902) = 0.9125\]