Section 2.4

3. (a) $\mu_X = 1(0.4) + 2(0.2) + 3(0.2) + 4(0.1) + 5(0.1) = 2.3$

(b) $\sigma_X^2 = (1 - 2.3)^2(0.4) + (2 - 2.3)^2(0.2) + (3 - 2.3)^2(0.2) + (4 - 2.3)^2(0.1) + (5 - 2.3)^2(0.1) = 1.81$
   Alternatively, $\sigma_X^2 = 1^2(0.4) + 2^2(0.2) + 3^2(0.2) + 4^2(0.1) + 5^2(0.1) - 2.3^2 = 1.81$

(c) $\sigma_X = \sqrt{1.81} = 1.345$

(d) $Y = 10X$. Therefore the probability density function is as follows.

<table>
<thead>
<tr>
<th>y</th>
<th>p(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.4</td>
</tr>
<tr>
<td>20</td>
<td>0.2</td>
</tr>
<tr>
<td>30</td>
<td>0.2</td>
</tr>
<tr>
<td>40</td>
<td>0.1</td>
</tr>
<tr>
<td>50</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(e) $\mu_Y = 10(0.4) + 20(0.2) + 30(0.2) + 40(0.1) + 50(0.1) = 23$

(f) $\sigma_Y^2 = (10 - 23)^2(0.4) + (20 - 23)^2(0.2) + (30 - 23)^2(0.2) + (40 - 23)^2(0.1) + (50 - 23)^2(0.1) = 181$
   Alternatively, $\sigma_Y^2 = 10^2(0.4) + 20^2(0.2) + 30^2(0.2) + 40^2(0.1) + 50^2(0.1) - 23^2 = 181$

(g) $\sigma_Y = \sqrt{181} = 13.45$

5. (a) 

<table>
<thead>
<tr>
<th>x</th>
<th>p(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.70</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>0.02</td>
</tr>
</tbody>
</table>

(b) $P(X \leq 2) = P(X = 1) + P(X = 2) = 0.70 + 0.15 = 0.85$

(c) $P(X > 3) = P(X = 4) + P(X = 5) = 0.03 + 0.02 = 0.05$

(d) $\mu_X = 1(0.70) + 2(0.15) + 3(0.10) + 4(0.03) + 5(0.02) = 1.52$

(e) $\sigma_X = \sqrt{1^2(0.70) + 2^2(0.15) + 3^2(0.10) + 4^2(0.03) + 5^2(0.02) - 1.52^2} = 0.9325$

7. (a) $\sum_{x=1}^{4} cx = 1$, so $c(1 + 2 + 3 + 4) = 1$, so $c = 0.1$.

(b) $P(X = 2) = c(2) = 0.1(2) = 0.2$

(c) $\mu_X = \sum_{x=1}^{4} xP(X = x) = \sum_{x=1}^{4} 0.1x^2 = (0.1)(1^2 + 2^2 + 3^2 + 4^2) = 3.0$

(d) $\sigma_X^2 = \sum_{x=1}^{4} (x - \mu_X)^2P(X = x) = \sum_{x=1}^{4} (x - 3)^2(0.1x) = 4(0.1) + 1(0.2) + 0(0.3) + 1(0.4) = 1$
   Alternatively, $\sigma_X^2 = \sum_{x=1}^{4} x^2P(X = x) - \mu_X^2 = \sum_{x=1}^{4} 0.1x^3 - 3^2 = 0.1(1^3 + 2^3 + 3^3 + 4^3) - 3^2 = 1$

(e) $\sigma_X = \sqrt{1} = 1$
13. (a) \[ \int_{80}^{90} \frac{x - 80}{800} \, dx = \frac{x^2 - 160x}{1600} \bigg|_{80}^{90} = 0.0625 \]

(b) \[ \int_{80}^{120} \frac{x - 80}{800} \, dx = \frac{x^3 - 120x}{2400} \bigg|_{80}^{120} = \frac{320}{3} = 106.67 \]

(c) \[ \sigma_X^2 = \int_{80}^{120} \frac{x^3 - 80}{800} \, dx - \left(\frac{320}{3}\right)^2 = \frac{x^4}{3200} - \frac{x^3}{30} \bigg|_{80}^{120} - \left(\frac{320}{3}\right)^2 = 800/9 \]

\[ \sigma_X = \sqrt{800/9} = 9.428 \]

(d) \[ F(x) = \int_{-\infty}^{x} f(t) \, dt \]

If \( x < 80 \), \( F(x) = \int_{-\infty}^{80} 0 \, dt = 0 \)

If \( 80 \leq x < 120 \), \( F(x) = \int_{-\infty}^{80} 0 \, dt + \int_{80}^{x} \frac{t - 80}{800} \, dt = x^2 / 1600 - x / 10 + 4 \)

If \( x \geq 120 \), \( F(x) = \int_{-\infty}^{80} 0 \, dt + \int_{80}^{120} \frac{t - 80}{800} \, dt + \int_{120}^{x} 0 \, dt = 1 \).

21. (a) \[ P(X > 0.5) = \int_{0.5}^{1} 1.2(x + x^2) \, dx = 0.6x^2 + 0.4x^3 \bigg|_{0.5}^{1} = 0.8 \]

(b) \[ \mu = \int_{0}^{1} 1.2x(x + x^2) \, dx = 0.4x^3 + 0.3x^4 \bigg|_{0}^{1} = 0.7 \]

(c) \( X \) is within \pm 0.1 of the mean if \( 0.6 < X < 0.8 \).

\[ P(0.6 < X < 0.8) = \int_{0.6}^{0.8} 1.2(x + x^2) \, dx = 0.6x^2 + 0.4x^3 \bigg|_{0.6}^{0.8} = 0.2864 \]

(d) The variance is

\[ \sigma^2 = \int_{0}^{1} 1.2x^2(x + x^2) \, dx - \mu^2 \]

\[ = 0.3x^4 + 0.24x^5 \bigg|_{0}^{1} - 0.7^2 \]

\[ = 0.05 \]

The standard deviation is \( \sigma = \sqrt{0.05} = 0.2236 \).
(e) \( X \) is within \( \pm 2\sigma \) of the mean if \( 0.2528 < X < 1.1472 \). Since \( P(X > 1) = 0 \), \( X \) is within \( \pm 2\sigma \) of the mean if \( 0.2528 < X < 1 \).

\[
P(0.2528 < X < 1) = \int_{0.2528}^{1} 1.2(x + x^2) dx = 0.6x^2 + 0.4x^3 \bigg|_{0.2528}^{1} = 0.9552
\]

(f) \( F(x) = \int_{-\infty}^{x} f(t) dt \)

- If \( x < 0 \), \( F(x) = \int_{-\infty}^{x} 0 dt = 0 \)
- If \( 0 < x < 1 \), \( F(x) = \int_{0}^{x} 1.2(t + t^2) dt = 0.6x^2 + 0.4x^3 \)
- If \( x > 1 \), \( F(x) = \int_{0}^{1} 1.2(t + t^2) dt = 1 \)

Section 2.5

1. (a) \( \mu_X = 3\mu_X = 3(9.5) = 28.5 \)
   \( \sigma_X = 3\sigma_X = 3(0.4) = 1.2 \)

(b) \( \mu_{Y-X} = \mu_Y - \mu_X = 6.8 - 9.5 = -2.7 \)
   \( \sigma_{Y-X} = \sqrt{\sigma_Y^2 + \sigma_X^2} = \sqrt{0.1^2 + 0.4^2} = 0.412 \)

(c) \( \mu_{X+4Y} = \mu_X + 4\mu_Y = 9.5 + 4(6.8) = 36.7 \)
   \( \sigma_{X+4Y} = \sqrt{\sigma_X^2 + 4^2\sigma_Y^2} = \sqrt{0.4^2 + 16(0.1^2)} = 0.566 \)

3. Let \( X_1, \ldots, X_5 \) be the lifetimes of the five bulbs. Let \( S = X_1 + \ldots + X_5 \) be the total lifetime.

\[
\mu_S = \sum \mu_{X_i} = 5(700) = 3500 \\
\sigma_S = \sqrt{\sum \sigma_{X_i}^2} = \sqrt{5(20^2)} = 44.7
\]

15. (a) \( P(X < 9.98) = \int_{9.95}^{9.98} 10 \, dx = 10x \bigg|_{9.95}^{9.98} = 0.3 \)

(b) \( P(Y > 5.01) = \int_{5.01}^{5.1} 5 \, dy = 5y \bigg|_{5.01}^{5.1} = 0.45 \)

(c) Since \( X \) and \( Y \) are independent,
\[ P(X < 9.98 \text{ and } Y > 5.01) = P(X < 9.98)P(Y > 5.01) = (0.3)(0.45) = 0.135 \]
(d) \( \mu_X = \int_{9.95}^{10.05} 10x \, dx = 5x^2 \bigg|_{9.95}^{10.05} = 10 \)

(e) \( \mu_Y = \int_{4.9}^{5.1} 5y \, dy = 2.5y^2 \bigg|_{4.9}^{5.1} = 5 \)