Exam: I
Time: 1 hour 15 minutes
Date: 03/03/2009
Name: SAMPLE

Please read carefully following information about Exam I

1. Useful formulas will be provided.
2. Will cover from Chapter 23 to Chapter 28 of textbook.
4. Please review Quiz I (key is uploaded in the course webpage).
5. Please review all of the solved problems in the book (examples given in textbook).
6. Questions will be both multiple choices as well as problems requiring long answers.

Few examples of long questions:
1. Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V.

2. Calculate the speed of an electron that is accelerated through the same potential difference.

\[
K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f
\]

or,
\[
0 + qV + 0 = \frac{1}{2} m_p V_p^2 + 0
\]

or,
\[
(1.6 \times 10^{-19})(120) = \frac{1}{2} (1.67 \times 10^{-27}) V_p^2
\]

\[
\Rightarrow V_p = 1.52 \times 10^5 \text{ m/s}
\]

The proton will gain speed in moving the other way from \( V_i = 0 \) to \( V_f = 120 \text{ V} \).

\[
K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f
\]

or,
\[
0 + 0 + 0 = \frac{1}{2} m_e V_e^2 + qV
\]

or,
\[
0 = \frac{1}{2} (9.11 \times 10^{-31}) V_e^2 + (-1.6 \times 10^{-19})(120)
\]

\[
\Rightarrow V_e = 6.49 \times 10^6 \text{ m/s}
\]
2. Find the equivalent capacitance between points a and b for a group of capacitors connected as shown below: 

Take, \(C_1 = 5 \text{ \mu F}, \quad C_2 = 10 \text{ \mu F}, \quad C_3 = 2 \text{ \mu F}\).

**Step 1:** First consider \(C_1\) and \(C_2\) only. \(C_1\) and \(C_2\) are in series.

So,
\[
C_s = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{1}{5} + \frac{1}{10} \right)^{-1} = 3.33 \text{ \mu F}
\]

Since there are two halves in the combination of \(C_1\) & \(C_2\), each of them makes parallel connection with \(C_3\) as shown below:

Then, equivalent capacitance of upper parts is,
\[
C_1 = \frac{1}{2} C_s + C_3 = \frac{2}{3.33} + 2 = 6.66 \text{ \mu F}
\]

For lower part,
\[
C_2 = C_3 + C_3 = 2 + 2 = 4 \text{ \mu F}
\]

Finally, the equivalent capacitance between points a and b is given by,
\[
C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{1}{6.66} + \frac{1}{4} \right)^{-1} = 2.77 \text{ \mu F}
\]
3. Four resistors are connected as shown in the figure below.
(a) Find the equivalent resistance between points a and c.
(b) What is the current in each resistor if a potential difference of 42 V is maintained between a and c.

\[ R_{eq} = R_1 + R_2 = 8 + 4 = 12 \Omega \]

\[ R_{eq} = \left( \frac{1}{R_3} + \frac{1}{R_4} \right)^{-1} = \left( \frac{1}{6} + \frac{1}{3} \right)^{-1} = 2 \Omega \]

Hence, equivalent resistance.

\[ R_{eq} = R_{eq1} + R_{eq2} = 12 + 2 = 14 \Omega \]

(b) Since, \( \Delta V_{ac} = 42 \) V and from part a, \( R_{eq} = 14 \Omega \)

So, \( I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42}{14} = 3 \) A

Hence, current through \( R_1 \) and \( R_2 \) will be same (since they are in series) and equal to 3 A.
At point b, I is divided in two parts \( I_1 \) & \( I_2 \).
But voltage across \( R_3 \) and \( R_4 \) is same.

So, \( \Delta V_3 = \Delta V_4 \Rightarrow R_3 \cdot I_1 = R_4 \cdot I_2 \)

or \( 6 \cdot I_1 = 3 \cdot I_2 \Rightarrow I_2 = 2I_1 \)

But, \( I = I_1 + I_2 \)

or \( 3 = I_1 + 2I_1 \)

or \( I_1 = 1 \) A

So, current through \( R_3 = I_1 \) A

Current through \( R_4 = I_2 = 2I_1 = 2 \) A.
4. Consider the combination of capacitors as shown in Figure below.

(a) What is the equivalent capacitance of the group?
(b) Determine the charge on each capacitor.

\[ \text{Smt:} \]

\[ \begin{array}{c}
   \begin{array}{c}
   \text{36V}
   \end{array}
   \begin{array}{c|c|c|c|c|c|c|c|c|c}
   4 \mu F & 0 \mu F & 2 \mu F & 8 \mu F & 2 \mu F & 3 \mu F & 6 \mu F & 8 \mu F & 24 \mu F
   \end{array}
   \end{array} \]

\[ \text{Figure 1} \quad \text{Figure 2} \quad \text{Figure 3} \]

(c) As explained in problem no. 2, the combination reduces to an equivalent capacitance of \(12 \mu F\) in stages as shown below.

\[ \begin{array}{c}
   \begin{array}{c}
   \text{36V}
   \end{array}
   \begin{array}{c|c|c|c|c|c|c|c|c|c}
   4 \mu F & 8 \mu F & 2 \mu F & 8 \mu F & 2 \mu F & 3 \mu F & 6 \mu F & 8 \mu F & 24 \mu F
   \end{array}
   \end{array} \]

\[ \text{Figure 1} \quad \text{Figure 2} \quad \text{Figure 3} \]

(b) From Figure 2,

Charge on \(C_1\), \(Q_1 = C_1 V = (9)(36) = 324 \mu C\)

Charge on \(C_2\), \(Q_2 = C_2 V = (2)(36) = 72 \mu C\)

Charge on equivalent capacitance of \(C_3\) & \(C_4\),

\[ Q_{34} = C_{eq} V \]

\[ = \left( \frac{1}{C_3} + \frac{1}{C_4} \right)(36) \]

\[ = \left( \frac{1}{24} + \frac{1}{8} \right)(36) \]

\[ = (6)(36) = 216 \mu C \]

Hence, Charge on \(C_3 = \text{Charge on } C_4 \)

\[ = 216 \mu C \]
5. Two resistors, A and B, are connected in parallel across a 6.0 V battery. The current through B is found to be 2.0 A. When the two resistors are connected in series to the 6.0 V battery, a voltmeter connected across resistor A measures a voltage of 4.0 V. Find the resistances of A and B.

**Solution:** First consider the parallel case. The resistance of resistor B is:

\[ R_B = \frac{(\Delta V)_B}{I_B} = \frac{6.0 \text{ V}}{2.0 \text{ A}} = 3.0 \Omega \]

In the series combination, the potential difference across B is given by:

\[ (\Delta V)_B - (\Delta V)_{\text{battery}} - (\Delta V)_A = 6.0 \text{ V} - 4.0 \text{ V} = 2.0 \text{ V} \]

The current through the series combination is then:

\[ I_S = \frac{(\Delta V)_B}{R_B} = \frac{2.0 \text{ V}}{3.0 \Omega} = \frac{2}{3} \text{ A} \]

And the resistance of resistor A is:

\[ R_A = \frac{(\Delta V)_A}{I_S} = \frac{4.0 \text{ V}}{\frac{2}{3} \text{ A}} = 6.0 \Omega \]