Chapter 29

Magnetic Fields
A Brief History of Magnetism

- **13\(^{th}\) century BC**
  - Chinese used a compass
    - Uses a magnetic needle
    - Probably an invention of Arabic or Indian origin

- **800 BC**
  - Greeks
    - Discovered magnetite \((\text{Fe}_3\text{O}_4)\) attracts pieces of iron
Pierre de Maricourt found that the direction of a needle near a spherical natural magnet formed lines that encircled the sphere. The lines also passed through two points diametrically opposed to each other. He called the points poles.
A Brief History of Magnetism, 3

1600

- William Gilbert
  - Expanded experiments with magnetism to a variety of materials
  - Suggested the Earth itself was a large permanent magnet
A Brief History of Magnetism, 4

- 1819
  - Hans Christian Oersted
    - Discovered the relationship between electricity and magnetism
  - An electric current in a wire deflected a nearby compass needle
A Brief History of Magnetism, final

● 1820’s
  ● Faraday and Henry
    ● Further connections between electricity and magnetism
    ● A changing magnetic field creates an electric field
  ● Maxwell
    ● A changing electric field produces a magnetic field
Magnetic Poles

- Every magnet, regardless of its shape, has two poles
  - Called north and south poles
  - Poles exert forces on one another
    - Similar to the way electric charges exert forces on each other
    - Like poles repel each other
      - N-N or S-S
    - Unlike poles attract each other
      - N-S
Magnetic Poles, cont.

- The poles received their names due to the way a magnet behaves in the Earth’s magnetic field
- If a bar magnet is suspended so that it can move freely, it will rotate
  - The magnetic north pole points toward the Earth’s north geographic pole
    - This means the Earth’s north geographic pole is a magnetic south pole
    - Similarly, the Earth’s south geographic pole is a magnetic north pole
Magnetic Poles, final

- The force between two poles varies as the inverse square of the distance between them.

- A single magnetic pole has never been isolated.
  - In other words, magnetic poles are always found in pairs.
  - All attempts so far to detect an isolated magnetic pole has been unsuccessful.
    - No matter how many times a permanent magnetic is cut in two, each piece always has a north and south pole.
Magnetic Fields

- Reminder: an electric field surrounds any electric charge
- The region of space surrounding any *moving* electric charge also contains a magnetic field
- A magnetic field also surrounds a magnetic substance making up a permanent magnet
Magnetic Fields, cont.

- A vector quantity
- Symbolized by \( \mathbf{B} \)
- Direction is given by the direction a north pole of a compass needle points in that location
- Magnetic field lines can be used to show how the field lines, as traced out by a compass, would look
Magnetic Field Lines, Bar Magnet Example

- The compass can be used to trace the field lines
- The lines outside the magnet point from the North pole to the South pole
Magnetic Field Lines, Bar Magnet

- Iron filings are used to show the pattern of the electric field lines.
- The direction of the field is the direction a north pole would point.
Magnetic Field Lines, Unlike Poles

- Iron filings are used to show the pattern of the electric field lines.
- The direction of the field is the direction a north pole would point.
  - Compare to the electric field produced by an electric dipole.
Magnetic Field Lines, Like Poles

- Iron filings are used to show the pattern of the electric field lines.
- The direction of the field is the direction a north pole would point.
  - Compare to the electric field produced by like charges.
Definition of Magnetic Field

- The magnetic field at some point in space can be defined in terms of the magnetic force, $F_B$
- The magnetic force will be exerted on a charged particle moving with a velocity, $\mathbf{v}$
  - Assume (for now) there are no gravitational or electric fields present
Force on a Charge Moving in a Magnetic Field

- The magnitude $F_B$ of the magnetic force exerted on the particle is proportional to the charge, $q$, and to the speed, $v$, of the particle.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle’s velocity vector makes any angle $\theta \neq 0$ with the field, the force acts in a direction perpendicular to both the velocity and the field.
\( \mathbf{F_B} \) on a Charge Moving in a Magnetic Field, final

- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.
- The magnitude of the magnetic force is proportional to \( \sin \theta \), where \( \theta \) is the angle the particle’s velocity makes with the direction of the magnetic field.
More About Direction

- \( \mathbf{F}_B \) is perpendicular to the plane formed by \( \mathbf{v} \) and \( \mathbf{B} \).
- Oppositely directed forces exerted on oppositely charged particles will cause the particles to move in opposite directions.
The properties can be summarized in a vector equation:

\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \]

- \( \mathbf{F}_B \) is the magnetic force
- \( q \) is the charge
- \( \mathbf{v} \) is the velocity of the moving charge
- \( \mathbf{B} \) is the magnetic field
Direction: Right-Hand Rule #1

- The fingers point in the direction of $\mathbf{v}$
- $\mathbf{B}$ comes out of your palm
  - Curl your fingers in the direction of $\mathbf{B}$
- The thumb points in the direction of $\mathbf{v} \times \mathbf{B}$ which is the direction of $\mathbf{F}_B$
Direction: Right-Hand Rule #2

- Alternative to Rule #1
- Thumb is in the direction of $\vec{v}$
- Fingers are in the direction of $\vec{B}$
- Palm is in the direction of $\vec{F}_B$
  - On a positive particle
  - You can think of this as your hand pushing the particle
More About Magnitude of F

- The magnitude of the magnetic force on a charged particle is $F_B = |q| \, v \, B \, \sin \theta$
  - $\theta$ is the smaller angle between $v$ and $B$
  - $F_B$ is zero when the field and velocity are parallel or antiparallel
    - $\theta = 0$ or $180^\circ$
  - $F_B$ is a maximum when the field and velocity are perpendicular
    - $\theta = 90^\circ$
Differences Between Electric and Magnetic Fields

- **Direction of force**
  - The electric force acts along the direction of the electric field
  - The magnetic force acts perpendicular to the magnetic field

- **Motion**
  - The electric force acts on a charged particle regardless of whether the particle is moving
  - The magnetic force acts on a charged particle only when the particle is in motion
More Differences Between Electric and Magnetic Fields

- Work
  - The electric force does work in displacing a charged particle
  - The magnetic force associated with a steady magnetic field does no work when a particle is displaced
    - This is because the force is perpendicular to the displacement
Work in Fields, cont.

- The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone.
- When a charged particle moves with a given velocity through a magnetic field, the field can alter the direction of the velocity, but not the speed or the kinetic energy.
Units of Magnetic Field

- The SI unit of magnetic field is the tesla (T)

\[ T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m} \]

- Wb is a weber
- A non-SI commonly used unit is a gauss (G)
  - $1 \, T = 10^4 \, G$
Notation Notes

- When vectors are perpendicular to the page, dots and crosses are used
  - The dots represent the arrows coming out of the page
  - The crosses represent the arrows going into the page
Charged Particle in a Magnetic Field

- Consider a particle moving in an external magnetic field with its velocity perpendicular to the field.
- The force is always directed toward the center of the circular path.
- The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle.
Force on a Charged Particle

- Equating the magnetic and centripetal forces:
  \[ F_B = qvB = \frac{mv^2}{r} \]

- Solving for \( r \):
  \[ r = \frac{mv}{qB} \]

- \( r \) is proportional to the linear momentum of the particle and inversely proportional to the magnetic field.
More About Motion of Charged Particle

- The angular speed of the particle is
  \[ \omega = \frac{v}{r} = \frac{qB}{m} \]

- The angular speed, \( \omega \), is also referred to as the cyclotron frequency.

- The period of the motion is
  \[ T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} \]
Motion of a Particle, General

- If a charged particle moves in a magnetic field at some arbitrary angle with respect to the field, its path is a helix
- Same equations apply, with

\[ v_z = \sqrt{v_x^2 + v_y^2} \]
Bending of an Electron Beam

- Electrons are accelerated from rest through a potential difference
- The electrons travel in a curved path
- Conservation of energy will give $v$
- Other parameters can be found
Particle in a Nonuniform Magnetic Field

- The motion is complex
- For example, the particles can oscillate back and forth between two positions
- This configuration is known as a magnetic bottle
Van Allen Radiation Belts

- The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions.
- The particles are trapped by the Earth’s magnetic field.
- The particles spiral from pole to pole.
  - May result in Auroras.
Charged Particles Moving in Electric and Magnetic Fields

- In many applications, charged particles will move in the presence of both magnetic and electric fields.
- In that case, the total force is the sum of the forces due to the individual fields.
- In general: \[ \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \]
Velocity Selector

- Used when all the particles need to move with the same velocity.
- A uniform electric field is perpendicular to a uniform magnetic field.
Velocity Selector, cont.

- When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line.
- This occurs for velocities of value
  \[ v = \frac{E}{B} \]
Velocity Selector, final

- Only those particles with the given speed will pass through the two fields undeflected.
- The magnetic force exerted on particles moving at speed greater than this is stronger than the electric field and the particles will be deflected to the left.
- Those moving more slowly will be deflected to the right.
Mass Spectrometer

- A mass spectrometer separates ions according to their mass-to-charge ratio.
- A beam of ions passes through a velocity selector and enters a second magnetic field.
Mass Spectrometer, cont.

- After entering the second magnetic field, the ions move in a semicircle of radius \( r \) before striking a detector at \( P \)
- If the ions are positively charged, they deflect to the left
- If the ions are negatively charged, they deflect to the right
Thomson’s $e/m$ Experiment

- Electrons are accelerated from the cathode
- They are deflected by electric and magnetic fields
- The beam of electrons strikes a fluorescent screen
- $e/m$ was measured
Cyclotron

- A **cyclotron** is a device that can accelerate charged particles to very high speeds.
- The energetic particles produced are used to bombard atomic nuclei and thereby produce reactions.
- These reactions can be analyzed by researchers.
Cyclotron, 2

- $D_1$ and $D_2$ are called dees because of their shape.
- A high frequency alternating potential is applied to the dees.
- A uniform magnetic field is perpendicular to them.
Cyclotron, 3

- A positive ion is released near the center and moves in a semicircular path.
- The potential difference is adjusted so that the polarity of the dees is reversed in the same time interval as the particle travels around one dee.
- This ensures the kinetic energy of the particle increases each trip.
Cyclotron, final

- The cyclotron’s operation is based on the fact that $T$ is independent of the speed of the particles and of the radius of their path.

$$K = \frac{1}{2} m v^2 = \frac{q^2 B^2 R^2}{2m}$$

- When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play.
Magnetic Force on a Current Carrying Conductor

- A force is exerted on a current-carrying wire placed in a magnetic field
  - The current is a collection of many charged particles in motion
- The direction of the force is given by the right-hand rule
**Force on a Wire**

- In this case, there is no current, so there is no force
- Therefore, the wire remains vertical
Force on a Wire (2)

- The magnetic field is into the page
- The current is up the page
- The force is to the left
Force on a Wire, (3)

- The magnetic field is into the page
- The current is down the page
- The force is to the right
Force on a Wire, equation

- The magnetic force is exerted on each moving charge in the wire
  \[ \vec{F} = q \vec{v}_d \times \vec{B} \]
- The total force is the product of the force on one charge and the number of charges
  \[ \vec{F} = \left( q \vec{v}_d \times \vec{B} \right) nAL \]
Force on a Wire, (4)

- In terms of the current, this becomes
  \[ \vec{F}_B = I \vec{L} \times \vec{B} \]
  - \( I \) is the current
  - \( \vec{L} \) is a vector that points in the direction of the current
    - Its magnitude is the length \( L \) of the segment
  - \( \vec{B} \) is the magnetic field
Force on a Wire, Arbitrary Shape

- Consider a small segment of the wire, $d\vec{s}$
- The force exerted on this segment is $d\vec{F}_B = I \, d\vec{s} \times \vec{B}$
- The total force is $\vec{F}_B = I \int_a^b \, d\vec{s} \times \vec{B}$
Torque on a Current Loop

- The rectangular loop carries a current $I$ in a uniform magnetic field.
- No magnetic force acts on sides 1 & 3.
  - The wires are parallel to the field and $\mathbf{L} \times \mathbf{B} = 0$.
There is a force on sides 2 & 4 since they are perpendicular to the field.

The magnitude of the magnetic force on these sides will be:

\[ F_2 = F_4 = IaB \]

The direction of \( F_2 \) is out of the page.

The direction of \( F_4 \) is into the page.
Torque on a Current Loop, 3

- The forces are equal and in opposite directions, but not along the same line of action.
- The forces produce a torque around point $O$. 

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Torque on a Current Loop, Equation

- The maximum torque is found by:
  \[ \tau_{\text{max}} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB) \frac{b}{2} + (IaB) \frac{b}{2} \]
  \[ = IaB \]
- The area enclosed by the loop is \(ab\), so \(\tau_{\text{max}} = IAB\)
  - This maximum value occurs only when the field is parallel to the plane of the loop
Torque on a Current Loop, General

- Assume the magnetic field makes an angle of $\theta < 90^\circ$ with a line perpendicular to the plane of the loop.
- The net torque about point $O$ will be $\tau = IAB \sin \theta$. 
Torque on a Current Loop, Summary

- The torque has a maximum value when the field is perpendicular to the normal to the plane of the loop.
- The torque is zero when the field is parallel to the normal to the plane of the loop.
- \( \tau = I \hat{A} \times \hat{B} \) where \( \hat{A} \) is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop.
Direction

- The right-hand rule can be used to determine the direction of $\vec{A}$
- Curl your fingers in the direction of the current in the loop
- Your thumb points in the direction of $\vec{A}$
Magnetic Dipole Moment

- The product $I \hat{A}$ is defined as the magnetic dipole moment, $\mu$, of the loop
  - Often called the magnetic moment
- SI units: A · m²
- Torque in terms of magnetic moment:
  $$\hat{r} = \mu \times \hat{B}$$
  - Analogous to $\hat{r} = p \times \hat{E}$ for electric dipole
Potential Energy

- The potential energy of the system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field:
  \[ U = -\mu \cdot \mathbf{g} \cdot \mathbf{B} \]

- \( U_{\text{min}} = -\mu B \) and occurs when the dipole moment is in the same direction as the field.

- \( U_{\text{max}} = +\mu B \) and occurs when the dipole moment is in the direction opposite the field.
Hall Effect

- When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.
- This phenomena is known as the Hall effect.
- It arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic forces they experience.
Hall Effect, cont.

- The Hall effect gives information regarding the sign of the charge carriers and their density.
- It can also be used to measure magnetic fields.
Hall Voltage

- This shows an arrangement for observing the Hall effect
- The Hall voltage is measured between points $a$ and $c$
Hall Voltage, cont

- When the charge carriers are negative, the upper edge of the conductor becomes negatively charged
  - $c$ is at a lower potential than $a$
- When the charge carriers are positive, the upper edge becomes positively charged
  - $c$ is at a higher potential than $a$
Hall Voltage, final

- \( \Delta V_H = E_H d = v_d B d \)
  - \( d \) is the width of the conductor
  - \( v_d \) is the drift velocity
  - If \( B \) and \( d \) are known, \( v_d \) can be found

- \( \Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t} \)
  - \( R_H = 1 / nq \) is called the Hall coefficient
  - A properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field