1. A pulley which consists of a solid disk with 7 kg mass and 20 cm. radius is initially spinning at 40 rad/sec. It has a string attached to it and the string is attached to a 1.5 kg. mass. As the string is wound around the pulley it raises the weight from the ground. When it has lifted the mass 2 meters from the ground
a) what is the speed of the mass and the angular speed of the pulley? Assume no loss of energy when the string “jerks” the mass up to speed.

\[ W_i = \frac{1}{2} m v_i^2 + \frac{1}{2} I w_i^2 = m g h_2 + \frac{1}{2} m v_2^2 + \frac{1}{2} I w_2^2 \]

\[ v_i = \frac{4d(2)}{18} \text{ m/sec} \]

\[ \theta = \frac{\pi}{2} \text{ rad} \]

\[ \theta = 2 \text{ meter} \]

\[ \theta = \frac{2}{\pi} = 0.63 \text{ rad} \]

\[ h_2 = \frac{160}{9.8(1.5)} = 9.9 \text{ meter} \]

2. If you have a piece of aluminum which weighs 2 kg. but only appears to weigh 1.65 kg. when submerged in an unknown liquid, find the density of the unknown liquid.

\[ m = 2 kg = \rho_{Al} \text{ Vol} \]

\[ \text{Vol} = \frac{2}{2.7 \times 10^{-3}} = 0.74 m^3 \]

\[ \text{Weight} - \text{Weight in fluid} = \rho_{fluid} \text{ Vol} \]

\[ 2(9.8) - 1.65(9.8) = \rho_{fluid} \text{ Vol} \]

\[ 2 - 1.65 = 0.35 = \rho_{fluid} \times (0.0074) \]

\[ \rho_{fluid} = \frac{0.35}{0.0074} = 47.3 \text{ kg/m}^3 \]
3. A 10 meter diameter, 10 meter tall tank has a 2 cm. pipe at the very bottom. The tank top is sealed, and the pressure at the top is only .9 atmospheres. If it filled with water,

a) what is the initial volume rate of flow (m³/sec) through the pipe?

\[ \rho g h_1 + \frac{1}{2} \rho v_1^2 + p_1 = \rho g h_2 + \frac{1}{2} \rho v_2^2 + p_2 \]

\[ 1000(9.8)(10) + \frac{1}{2} \rho v_1^2 + .9 \times 10^5 = 0 + \frac{1}{2} \rho v_2^2 + 1 \times 10^5 \]

\[ \rho = 1 \times 10^5 \quad \rho = .9 \times 10^5 \]

\[ v_1 < v_2 \quad \quad \quad v_1^2 \leq 0 \]

\[ 1000(9.8)(10) - .1 \times 10^5 = \frac{1}{2} (1000) v_2^2 \quad \quad v_2 = 15.3 \text{ m/sec} \]

\[ A_2 v_2 = \pi (0.1)^2 (13.3) \]

\[ 1000(9.8)(10) - .1 \times 10^5 = 1000(9.8) h_2 + 0 \quad \rightarrow \text{stops leaking} \]

\[ h_2 = 8.98 \text{ meter} \]

b) if the pipe is extended up the side of the tank, what is the maximum height it could be extended and still have some leakage?

\[ 1000(9.8)(10) - .1 \times 10^5 = 1000(9.8) h_2 + 0 \]

\[ h_2 = 8.98 \text{ meter} \]

4. 0.25 kg of copper at 450 °C (solid) is added to .5 kg of water at 15°C. Find the temperature and mixture at equilibrium.

\[ .25(387)(450-T_f) = .5(4186)(T_f-15) \]

\[ 43538 - 9685T_f = 20937T_f - 31395 \]

\[ T_f = 34.2°C \]

5. A man hangs from a spring which has a 5000 nt./meter spring constant. Over a period of 20 seconds he "oscillates" through 25 complete up and down cycles. What is his mass?

\[ f = \frac{25 \text{ cycles}}{20 \text{ seconds}} = 1.25 \text{ Hz} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ (1.25)^2 (4\pi^2) = \frac{1}{m} \]

\[ \frac{5000}{5000} \]

\[ m = 81 \text{ kg} \]
6. A stretched string and an air column which is closed at one end, open at the other are in resonance with each other. The air column is 40 cm long, filled with air at 0°C, and is vibrating at its fundamental frequency. The string is one meter long and has a total mass of 10 grams. (m/L is just 10 grams per meter) If the string is vibrating in it's 3rd harmonic frequency (and this matches the pipes fundamental)

a) what is the fundamental frequency of the vibrating air column in the pipe?

\[ V = f \lambda \] for \( \lambda \) in closed and pipe \( \frac{1}{4} \lambda = L \)

\[ 330 = f_1 \times 1.6 \]

\[ f_1 = 206 \text{ Hz} \]

\[ \lambda = 4L \]

\[ 1.6m \]

b) what is the speed of the wave in the string?

3rd harmonic \( \frac{3}{2} \lambda = L \)

\( \lambda = \frac{2}{3} L = \frac{2}{3} (1) = .67 \text{m} \)

\[ V = f_3 \lambda_3 = 206 (1.67) = 338 \text{ m/sec} \]

c) what is the tension in the string?

\[ V = \sqrt{\frac{T}{\mu}} \]

\[ 138 = \sqrt{\frac{T}{10 \times 10^{-3}}} \]

\[ (138)^2 (10 \times 10^{-3}) = T \]

\[ T = 191 \text{ newton} \]

7. A policeman aims his speed gun as a moving car. The "gun" emits a sonar signal instead of radar (the wave travels at 340 m/sec. as it is a sound wave, not a light wave)

The source sends out a signal at 500 Hz. The car has a receiver which "hears" the signal and it appears to be 555Hz. How fast is the car traveling if the police car is stationary?

\[ f_0 = 500 \text{ Hz} \]

\[ f = 555 \text{ Hz} \]

\[ f = f_0 \left( \frac{V}{V - V_s} \right) \rightarrow 555 = 500 \left( \frac{340}{340 - V_s} \right) \]

\[ \frac{555}{500} = \frac{340}{340 - V_s} \]

\[ 340 = 1.11 (340) - 1.11 V_s \]

\[ V_s = 33.7 \text{ m/sec} \]

\[ f = f_0 \left( \frac{V + V_o}{V} \right) \]

\[ 555 = 500 \left( \frac{340 + V_o}{340} \right) \]

\[ 555 (340) = 500 (340 + V_o) \]

\[ \frac{55 (340)}{500} = V_o = 37.4 \text{ m/sec} \]