28.11 The energy of the emitted photon is

\[
E_\gamma = \frac{hc}{\lambda} = \left(\frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{656 \text{ nm}}\right)\left(\frac{2.998 \times 10^8 \text{ m/s}}{1 \text{ nm}}\right)\left(\frac{1 \text{ ev}}{1.602 \times 10^{-19} \text{ J}}\right) = 1.89 \text{ eV}
\]

This photon energy is also the difference in the electron’s energy in its initial and final orbits. The energies of the electron in the various allowed orbits within the hydrogen atom are

\[
E_n = -\frac{13.6}{n^2} \text{ eV} \quad \text{where} \quad n = 1, 2, 3, \ldots
\]

giving \( E_1 = -13.6 \text{ eV}, E_2 = -3.40 \text{ eV}, E_3 = -1.51 \text{ eV}, E_4 = -0.850 \text{ eV}, \ldots \)

Observe that \( E_2 = E_3 - E_1 \). Thus, the transition was from the \( n = 3 \) orbit to the \( n = 2 \) orbit.

28.12 The change in the energy of the electron is

\[
\Delta E = E_\gamma - E_i = 13.6 \text{ eV} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)
\]

Transition I: \( \Delta E = 13.6 \text{ eV} \left(\frac{1}{4} - \frac{1}{25}\right) = 2.86 \text{ eV} \) (absorption)

Transition II: \( \Delta E = 13.6 \text{ eV} \left(\frac{1}{25} - \frac{1}{9}\right) = -0.967 \text{ eV} \) (emission)

Transition III: \( \Delta E = 13.6 \text{ eV} \left(\frac{1}{49} - \frac{1}{16}\right) = -0.572 \text{ eV} \) (emission)

Transition IV: \( \Delta E = 13.6 \text{ eV} \left(\frac{1}{16} - \frac{1}{49}\right) = 0.572 \text{ eV} \) (absorption)

(a) Since \( \lambda = \frac{hc}{E_\gamma} = \frac{hc}{-\Delta E} \), transition II emits the shortest wavelength photon.

(b) The atom gains the most energy in transition I

(c) The atom loses energy in transitions II and III
(b) Using $x_n = \frac{n^2 a_b}{Z}$ gives $x = \frac{(1)^2 a_b}{3} = \frac{0.0529 \times 10^{-9}}{3} = 1.76 \times 10^{-11}$ m.

28.28 (a) The energy levels of a hydrogen-like ion whose charge number is $Z$ are given by

$$E_n = \left( -13.6 \text{ eV} \right) \frac{Z^2}{n^2}$$

For Helium, $Z = 2$ and the energy levels are

$$E_n = \frac{-54A \text{ eV}}{n^2} \quad n = 1, 2, 3, \ldots$$

(b) For He$, Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state) is

$$E = E_\infty - E_1 = 0 - \left( -13.6 \text{ eV} \right) \frac{(2)^2}{(1)^2} = 54A \text{ eV}$$