P39.7  (a) \[ \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-0.500^2}} = \frac{2}{\sqrt{3}} \]

The time interval between pulses as measured by the Earth observer is
\[ \Delta t = \gamma \Delta t'_p = \frac{2}{\sqrt{3}} \left( \frac{60 \text{ s}}{75 \text{ s}} \right) = 0.924 \text{ s} . \]

Thus, the Earth observer records a pulse rate of \( \frac{60 \text{ s/m in}}{0.924 \text{ s}} = 64.9 \text{ m in} \).

(b) At a relative speed \( v = 0.990c \), the relativistic factor \( \gamma \) increases to 7.09 and the pulse rate recorded by the Earth observer decreases to \( \frac{10 \text{ s/m in}}{7.09} \). That is, the life span of the astronaut (reckoned by the duration of the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

P39.9 \[ \Delta t = \gamma \Delta t'_p = \frac{\Delta t'_p}{\sqrt{1-v^2/c^2}} \quad \text{so} \quad \Delta t'_p = \left( \frac{1-v^2}{c^2} \right) \Delta t \]

and \[ \Delta t - \Delta t'_p = \left( \frac{v^2}{2c^2} \right) \Delta t . \]

If \( v = 1000 \text{ km/h} = \frac{100 \times 10^3 \text{ m}}{3600 \text{ s}} = 277.8 \text{ m/s} \)
then \[ \frac{V}{c} = 9.26 \times 10^{-7} \]
and \[ (\Delta t - \Delta t'_p) = \left( 4.28 \times 10^{-13} \right) (3600 \text{ s}) = 1.54 \times 10^{-9} \text{ s} = 1.54 \text{ ns} . \]

P39.15 (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is \( 20 \text{ m} \).

(b) His ship is in motion relative to you, so you measure its length contracted to \( 19 \text{ m} \).

(c) We have \( L = L'_p \sqrt{1-\frac{v^2}{c^2}} \)
from which \( \frac{L}{L'_p} = \frac{19 \text{ m}}{20 \text{ m}} = 0.950 = \sqrt{1-\frac{v^2}{c^2}} \) and \( v = 0.312c \).
\[ \sum W = K_f - K_i = \left( \frac{1}{\sqrt{1 - (v_f/c)^2}} - 1 \right) m c^2 - \left( \frac{1}{\sqrt{1 - (v_i/c)^2}} \right) m c^2 \]

or \[ \sum W = \left( \frac{1}{\sqrt{1 - (v_f/c)^2}} - \frac{1}{\sqrt{1 - (v_i/c)^2}} \right) m c^2 \]

(a) \[ \sum W = \left( \frac{1}{\sqrt{1 - (0.750)^2}} - \frac{1}{\sqrt{1 - (0.500)^2}} \right) (1.673 \times 10^{-27} \text{ kg}) (2.998 \times 10^8 \text{ m/s})^2 \]

\[ \sum W = 5.37 \times 10^{-11} \text{ J} \]

(b) \[ \sum W = \left( \frac{1}{\sqrt{1 - (0.995)^2}} - \frac{1}{\sqrt{1 - (0.500)^2}} \right) (1.673 \times 10^{-27} \text{ kg}) (2.998 \times 10^8 \text{ m/s})^2 \]

\[ \sum W = 1.33 \times 10^{-9} \text{ J} \]

P39.38 (a) Using the classical equation,

\[ K = \frac{1}{2} m v^2 = \frac{1}{2} (78.0 \text{ kg}) (1.06 \times 10^5 \text{ m/s})^2 = 4.38 \times 10^{11} \text{ J} \]

(b) Using the relativistic equation,

\[ K = \left( \frac{1}{\sqrt{1 - (v/c)^2}} - 1 \right) m c^2 \]

\[ K = \left[ \frac{1}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} - 1 \right] (78.0 \text{ kg}) (2.998 \times 10^8 \text{ m/s})^2 = 4.38 \times 10^{11} \text{ J} \]

When \( \frac{v}{c} \ll 1 \), the binomial series expansion gives

\[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{- \frac{1}{2}} \approx 1 + \frac{1}{2} \left( \frac{v}{c} \right)^2 \]

Thus,

\[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{- \frac{1}{2}} - 1 \approx \frac{1}{2} \left( \frac{v}{c} \right)^2 \]
and the relativistic expression for kinetic energy becomes
\[ K \approx \frac{1}{2}\left(\frac{v}{c}\right)^2 m c^2 = \frac{1}{2} m v^2. \] That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results.

P39.38

(a) Using the classical equation,
\[ K = \frac{1}{2} m v^2 = \frac{1}{2} (78.0 \text{ kg}) (1.06 \times 10^5 \text{ m/s})^2 = 4.38 \times 10^{11} \text{ J}. \]

(b) Using the relativistic equation,
\[ K = \left( \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) m c^2. \]
\[ K = \left[ \frac{1}{\sqrt{1 - (1.06 \times 10^5 / 2.998 \times 10^8)^2}} - 1 \right] (78.0 \text{ kg}) (2.998 \times 10^8 \text{ m/s})^2 = 4.38 \times 10^{11} \text{ J}. \]

When \( \frac{v}{c} \ll 1 \), the binomial series expansion gives
\[ \left[ 1 - \left(\frac{v}{c}\right)^2 \right]^{-\frac{1}{2}} \approx 1 + \frac{1}{2} \left(\frac{v}{c}\right)^2. \]

Thus,
\[ \left[ 1 - \left(\frac{v}{c}\right)^2 \right]^{-\frac{1}{2}} - 1 \approx \frac{1}{2} \left(\frac{v}{c}\right)^2. \]

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