Z-scores for intervals:

Since almost all of our concepts lead to the standard normal distribution let’s consider one of the properties before continuing:

EX// I want to find the two cut off points that contain 95% of the z-scores:
Draw Picture

![Confidence Level: 95%](image)

\[ z_{\alpha/2} = -1.96 \]
\[ z = 0 \]
\[ z_{\alpha/2} = 1.96 \]

Notice I don’t want to look up .95, instead I want to look up .025 or .975.
What is find is the interval \((-1.96, 1.96)\) contains 95% of the Z scores \(\rightarrow P(-1.96 < Z < 1.96) = 0.95\)

PPSLIDE #1

EX// I want to find the two cut off points for the interval of 90%.

This time I would look up either .05 or .95.
The interval is:

We’ll come back to this concept shortly.

Confidence Interval for \(\mu\) with Known \(\sigma\):

Remember the **parameters** describe characteristics of a **population**: such as \((\mu, \sigma, p)\)

**Statistics** are describe characteristics of a **sample**: such as \(\bar{X}, s, \text{ and } \hat{p}\)

* \(\hat{p}\) is called the “sample proportion” of a Binomial. (Estimated probability of success – more after break)

If our samples are good, then our statistics can be used to estimate the parameters of the population (we’ve done that).

However, so far we’ve just found **point estimates**, meaning we have just found single values.

**For example**: If we look at the handout problem that we did in-class in chapter 3. This was dealing with the number of pets in 50 homes (show the handout). We found a **sample** mean of 2.2.
Assuming that the 50 houses was a good unbiased sample:
We can say that $\bar{X} = 2.2$ is a point estimate of the true, population mean $\mu$.

The next question should be: *How good is that estimate?*

We want to avoid simple saying the mean is 2.2.
Because if we were to take another sample of 50 houses, most likely we’d end up with a different value.

- So we’re going to use Confidence Intervals to determine the accuracy of our point estimates.
- We want to be able to say that this interval contains the true pop. mean, with a certain amount of confidence.

======

For all of our problems in this section assume that $X_1 \ldots X_n$ have mean $\mu$ and standard deviation $\sigma$ and either

1. that $X_1 \ldots X_n$ come from a normal distribution
2. or $n > 30$

Therefore, from chapter 6, we can assume that $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

And since $\bar{X} \sim$Normal, we can convert it to a standard normal:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Back to the beginning of lecture: **PPSLIDE#1**

- $P(1.96 < Z < 1.96) = 0.95$

- Using substitution we get: $P\left(-1.96 < \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} < 1.96\right) = .95$

- And after some algebra we get: $P\left(\bar{X} - 1.96*\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96*\frac{\sigma}{\sqrt{n}}\right) = .95$

- So the interval: $\left(\bar{X} - 1.96*\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96*\frac{\sigma}{\sqrt{n}}\right)$ is called a 95% confidence interval (CI) for $\mu$ it is associated with a .95 (or 95%) confidence level.

**INTERPRETATION:**
- We are 95% confident that this interval contains the true value of $\mu$.

**NOTE:** this is an interval for $\mu$ for when the $\sigma$ is known –when it isn’t we will have different formula

**EX/ page 327 #25**
In a study regarding the length of time students required to earn a bachelors degree, 80 students are randomly selected and are found to have a mean of 4.8 years. Assuming $\sigma$ is known to be 2.2 years, construct a 95% confidence interval estimate of the population mean.
• \( X \) = time until graduation  
• \( \mu \) = unknown  
• \( \sigma = 2.2 \)  
• Sample: \( n = 80 \)  
• \( \bar{X} = 4.8 \)

Find a 95% confidence interval for \( \mu \).

**USE:**  
\[
\left( \bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}} \right)
\]

\[
\left( 4.8 - 1.96 \frac{2.2}{\sqrt{80}}, 4.8 + 1.96 \frac{2.2}{\sqrt{80}} \right) = (4.8 - .482096, 4.8 + .482096) = (4.3, 5.3)
\]

**Interpretation:** We are 95% confident that the interval \([4.3, 5.3]\) contains the true value of \( \mu \).

**Note about notation and definition:**
• \([4.3, 5.3]\) is the **confidence interval** (CI)  
• 95% was our **confidence level** (C)  
• And .05 which is the complement of the confidence level is known as \( \alpha \)  
  - So, \( C = 1 - \alpha \) and \( \alpha = 1 - C \)  
  - Right now, \( \alpha \) doesn’t have a good name and it seems like an extra step. However, it’ll be used regularly as we go along.

Okay, what if we want to find a different confidence interval.
• Everything will stay the same except for the z-score.  
• Need to find a z-score corresponding to the desired confidence level.

**EX//** -- Find the z-score for a 90% confidence interval  
**Confidence level,** \( C = .9 \) \( \rightarrow \) \( \alpha = 0.10 \)

• Draw \( \rightarrow \) but with 90%  
• \( \alpha \) is the complement however it is cut in half.  
• So we are looking for the z-score of \( Z_{0.05} = -1.645 \)  
• *we could also find the other side.*
EX/– Find the z-score for a 99% confidence interval
Confidence level, \( C = .99 \Rightarrow \alpha = 0.01 \)
- Draw \( \Rightarrow \) but with 99%
- \( \alpha \) is the complement however it is cut in half.
- So we are looking for the z-score of:
  \( Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = -2.575 \)

*we could also find the other side...

EX/ page 327 #25 -again
Now find a 99% confidence interval.
- \( X \) = time until graduation
- \( \mu \) = unknown
- \( \sigma = 2.2 \)
- Sample: \( n = 80 \)
- \( \bar{X} = 4.8 \)

Find a 99% confidence interval for \( \mu \).

USE:
\[
\left( \bar{X} - 2.575 \frac{\sigma}{\sqrt{n}}, \bar{X} + 2.575 \frac{\sigma}{\sqrt{n}} \right) = \left( 4.8 - 2.575 \frac{2.2}{\sqrt{80}}, 4.8 + 2.575 \frac{2.2}{\sqrt{80}} \right) = (4.8 - .633366, 4.8 + .633366) = (4.2, 5.4)
\]

Notice that if we want a higher confidence that the interval has to get bigger.
- It has to include more possibilities for \( \mu \)
- You can have a CI that is 99.999% but your interval will be really large.
- It’s like saying “We are 99.9999% confident, that the mean is somewhere between 4 and 700”
- You have to know what is more important to you.

IN GENERAL:
To find a confidence interval for \( \mu \) with known \( \sigma \)
\[
\left( \bar{X} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)
\]
\[
\left( \bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)
\]

The book likes to use the notation: \( Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = E \), The Margin of Error

So another way of writing our confidence interval is: \( (\bar{X} - E, \bar{X} + E) \) \( o \bar{X} \pm E \)
When people smoke, the nicotine they absorb is converted to cotinine (coat-in-ene), which can be measured. A sample of 40 smokers has a mean cotinine level of 172.5. Assuming that \( \sigma \) is known to be 119.5, find a 90% confidence interval estimate of the mean cotinine level of all smokers.

- \( X = \) level of cotinine
- \( \mu = \) unknown
- \( \sigma = 119.5 \)
- Sample: \( n = 40 \)
- \( \bar{X} = 172.5 \)
- \( \alpha = .10 , \alpha/2 = .05 \) \( \Rightarrow Z_{\alpha/2} = Z.05 = 1.645 \)

\[
\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \text{where} \quad Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 1.645 \frac{119.5}{\sqrt{40}} = 31.08163
\]

\[
\bar{X} \pm 31.08163 \\
172.5 \pm 31.08163 \\
(141.4, 203.58)
\]

\textit{GROUP WORK 27 page 327}

55.4 < \mu < 61.2 \; \text{yes because the interval contains 60 seconds.}

### Choosing a sample size:

Determining a sample size is an important concept. If the sample size is too small we get misleading and poor results, if the sample size is too big you can waste time and money. Usually, what happens is that you choose a margin of error (E) that is acceptable. From there solve for the sample size:

The general confidence interval formula is:

\[
\bar{X} \pm E \quad E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}
\]

Solve for \( n \) we get:

\[
n = \left( \frac{z_{\alpha/2} \sigma}{E} \right)^2
\]

\textit{EX/ page 328 #36}

Media Research wants to estimate the mean amount of time (in minutes) that full-time college students spend watching TV each weekday. Find the sample size necessary to estimate the mean with a 15-min margin of error. Assume that a 96% confidence level is desired and that a pilot study showed that the standard deviation is estimated to be 112.2 min.

\[
\sigma = 112.2 \\
E= 15 \\
Z_{\alpha/2} = Z .02 = -2.05 \\
n = \left( \frac{2.05 \times 112.2}{15} \right)^2 = 235.13 = 236
\]

\textit{GROUP WORK #35 page 328}

\( n= 6907 \)

### Finding a confidence interval for \( \mu \) with unknown \( \sigma \):

Last time: We found the confidence intervals for \( \mu \) with a known standard deviation.
Now we will consider if we don’t know the population standard deviation, \( \sigma \). Either the sample standard deviation will be given to us or we will have to calculate it.

Recall the formulas for sample standard deviation.

\[
s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2} \quad \text{OR} \quad s = \sqrt{\frac{n \sum(x^2) - (\sum x)^2}{n(n-1)}}
\]

Since we are estimating \( \sigma \) we can no longer use the z-table and the normal distribution. Instead, we use a t-score and the t-distribution.

Before: known \( \sigma \)

\[\bar{X} \pm \frac{\sigma}{\sqrt{n}}\]

Now: unknown \( \sigma \)

\[\bar{X} \pm \frac{s}{\sqrt{n}}\]

t-distribution (sometimes called a Student t distribution).

- similar to the normal distribution but it has more deviation –more spread and variation.
- shape of the t-distribution depends on the size of the sample space (PPSLIDE #2)
- the larger the sample space, the closer it is to a normal distribution

Because the shape of the t-distribution depends on the sample size, when we go to look up the values for the t-score we not only need to know the percentage that we want be we also need to know the “degrees of freedom”.

Degrees of Freedom: The number of values that can vary after certain restrictions have been imposed on all data values. (PPSLIDE #3)

EX// If 10 students take a quiz with a mean of 80, we can freely assign values to the first 9 students. But the 10th student will have a determined score. We know the sum of all 10 has to be 800 so when we add up the first nine it’ll determine what the 10th one will be. And because the first 9 could be freely assigned we say that there are 9 degrees of freedom.

For applications in this section the degrees of freedom is simply the sample size minus one:

Degrees of freedom = \( n-1 \)

EX// Find the t-statistic for 90% confidence interval that has a sample size of \( n = 20 \).

- \( t_{0.5} = t_{0.10/2} = t_{0.05} \)
- with 19 degrees of freedom, \( t_{0.05} = 1.729 \)

EX// Find the t-statistic for 95% confidence interval that has a sample size of \( n = 120 \).

- \( t_{0.5} = t_{0.05/2} = t_{0.025} \)
- with 119 degrees of freedom (closest is 100), \( t_{0.025} = 1.984 \)

EX// Find the t-statistic for 99% confidence interval that has a sample size of \( n = 52 \).

- \( t_{0.5} = t_{0.05/2} = t_{0.005} \)
- with 51 degrees of freedom (closest is 50), \( t_{0.005} = 2.678 \)

EX// page #340 #18 #20a,b

Data set in the Appendix includes 106 body temperatures for which \( \bar{X} = 98.20 \) and \( s = .62 \). Using the sample statistics construct a 99% confidence interval estimate of the mean of the body temperature of all healthy humans.
Since the standard deviation is calculated and not given we have to use the t-distribution so: \[ \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \]

- \( \bar{X} = 98.20 \)
- \( s = .62 \) * how do we know this is a sample standard deviation?
- \( n = 106 \)
- d.o.f = 105
- 99% CI ⇒ \( \alpha = .01 \)
- \( t_{\alpha/2} = t_{.005} = 2.626 \)

\[
\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 98.2 \pm 2.626 \frac{.62}{\sqrt{106}} = 98.2 \pm .15814 = (98.04, 98.36)
\]

**Interpretation:** We are 99% confident that the interval (98.04, 98.36) contains the value of the true population mean.

It suggests that the true mean might be less than 98.6.

**EX// page #24** The world's smallest mammal is the bumblebee bat (they are about the size of a bumble bee). Listed below are the weights (in grams) from a sample of bats. Construct a 95% confidence interval estimate of their mean weight. Are the confidence interval limits very different from the limits of 1.56 and 1.87 that are found when assuming that \( \sigma \) is known to be 0.30 grams?

1.7 1.6 1.5 2.0 2.3 1.6 1.8 1.5 1.7 2.2 1.4 1.6 1.6 1.6

- Find the mean and standard deviation
- Can use some sort of technology, either calc or StatCrunch *show stat crunch [file on server to paste data]*

- \( \bar{X} = 1.71 \)
- \( s = .25875 \)
- \( n = 15 \)
- d.o.f = 14
- 95% CI ⇒ \( \alpha = .05 \)
- \( t_{\alpha/2} = t_{.025} = 2.145 \)

\[
\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 1.713 \pm 2.145 \frac{.25875}{\sqrt{15}} = 1.713 \pm .14328 = (1.5697, 1.856)
\]

**GROUP WORK: page 340**

#17 (3002, 3204) #19 (-3.5, 0.9) since 0 is included the difference could be nothing.
Recall the properties of a Binomial Distribution: \( X = \text{number of success} \) and \( X \sim \text{Bin}(n, p) \) then
- \( n = \text{number of trials} \)
- \( X = \text{number of success out of} \ n \ \text{trials} \)
- \( p = \text{probability of success} \) but we can also think of this as the population portion of successes
- \( q = \text{probability of failure or the population proportion of failure} \)

EX// Flipping a coin. The probability of tails is .5, the other way to think about it is the population proportion is .5. However, if we were to flip a coin 100 times it would be unlikely that we would get exactly 50 tails and 50 heads. Instead, what we would have would be a sample proportion

NOTATION:
The “sample proportion” = \( \hat{p} \)

\[
\hat{p} = \frac{\text{number of successes}}{n} = \frac{x}{n} \quad \text{And} \quad \hat{q} = 1 - \hat{p}
\]

So I flip a coin 100 times and let \( \hat{p} \) represent the sample proportion of tails.

Now repeat.
Flip the coin 100 again: Now I have \( \hat{p}_2 \)
Again, \( \hat{p}_3 \) Again, \( \hat{p}_4 \)

What I have is \( \hat{p}_1, \hat{p}_2, \hat{p}_3, \ldots, \hat{p}_n \), which is now a data set.

- Now, like the sample means, the \( \hat{p} \)'s become random variables.
- This indicates that \( \hat{p} \) has a distribution, a mean and a standard deviation.
- The mean \( \mu_{\hat{p}} = p \) and \( \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} \)

Recall from Chapter 6:
\( X \), the number of successes could be approximated by a normal (as long as \( n \) was big enough).
The same idea can be applied here:
\( \hat{p} \) can be approximated by a normal distribution as long as there is 5 success and 5 failure in the sample.

\[
\hat{p} \sim N\left(\mu_{\hat{p}}, \sigma_{\hat{p}}\right)
\]

Remember the format of our previous confidence intervals:
\[ \bar{X} \pm \frac{z_{a/2}}{\sqrt{n}} \quad \text{OR} \quad \bar{X} \pm \frac{t_{a/2}}{\sqrt{n}} \]

For proportion it looks similar (skipping over the proof of this):

\[ \hat{p} \pm z_{a/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \quad \text{OR} \quad \hat{p} \pm E \quad \text{where} \quad E = z_{a/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \]

**EX// #32 page 314.** An important issue facing Americans is the large number of medical malpractice lawsuits and the expenses they generates. In a study of 1228 randomly selected medical malpractice lawsuits, it is found that 856 of them were later dropped or dismissed. Construct a 99% confidence interval estimate of the proportion of medical malpractice lawsuits that are dropped or dismissed. Does it appear that the majority of such suits are dropped or dismissed?

\[ \hat{p} = \frac{856}{1228} = .6971 \]
\[ \hat{q} = \frac{372}{1228} = .3029 \]

\[ n = 1228 \]
\[ C = .99 \]
\[ \alpha = 1 - .99 = .01 \]
\[ z_{a/2} = z_{.005} = 2.575 \]

\[ .6971 \pm 2.575 \sqrt{.6971*.3029/1228} \]

\[ (.6633, .7309) \]

Are the majority dismissed? -sure.

**FINDING THE SAMPLE SIZE FOR SAMPLE PROPORTION.**

Like the previous section, it’s sometimes necessary to determine sample size when dealing with proportion problems.

Formula:

\[ n = \left( \frac{z^2}{\alpha^2} \right) \left( \frac{\hat{p}\hat{q}}{E^2} \right) \]

This assumes that we have an estimate for the proportion. However, sometimes we don’t.

In that case, we go with the worst case scenario both \( p \) and \( q = .5 \)

\[ n = \left( \frac{z^2}{\alpha^2} \right) \left( \frac{1/2}{E^2} \right) \quad \Rightarrow \quad n = \left( \frac{z^2}{4E^2} \right) \]

This formula for \( n \) gives of the maximum sample size needed no matter what the true value of \( p \).

**EX//#42 page 316** How many randomly selected songs purchases must be surveyed to determine the percentage that were obtained by downloading? Assume that we want to be 95% confident that the sample percentage is within one percentage point of the true population percentage of songs that are downloaded.
Don’t have an estimate for $p$ → Use $n = \left( \frac{z^2}{4E^2} \right)$

$\hat{p} = .5$

$\hat{q} = .5$

$C = .95$

$\alpha = 1 - .95 = .05$

$z_{\alpha/2} = z_{.05/2} = z_{.025} = 1.96$

$E = .01$

**GROUP WORK** page 314

#35) [.0267, .0376]

#41) $n = 4145$

---

### Confidence Intervals for $\sigma$

To obtain a confidence interval for $\sigma$ we need to introduce another distribution called chi-squared $\chi^2$.

The chi-squared distribution, like the t-distribution, changes depending on the sample size. So we will be using degrees of freedom again. However, unlike the t-distribution, it is not symmetric. [PPSLIDE #5]

[PPSLIDE #4]

A symmetric distribution is convenient, we only have to find value for a z-score (or t-score) because we know the other side will be the simply the positive or negative.

---

**Confidence Interval for $\sigma$:** [PPSLIDE #6]

\[
\left( \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}} \right) \quad = \quad \left( \sqrt{\frac{(n-1)s^2}{\chi^2_{R}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{L}}} \right)
\]

- Since the $\chi^2$ distribution is not symmetric we have to look up 2 separate scores for a confidence interval.
- The area we find will be to the **right** of the score.
- \( \chi_{\alpha/2}^2 = \chi_R^2 \) will be the larger value.
- \( \chi_{1-\alpha/2}^2 = \chi_L^2 \) will be the smaller value.
- Table on page 615.

EX// Find the critical values of the \( \chi^2 \) for a 95% confidence interval with sample size of 10.

C=.95 \( \Rightarrow \) \( \alpha = .05 \) \( \Rightarrow \) \( \alpha/2 = .025 \) \( \Rightarrow \) \( 1-\alpha/2 = .975 \)

\[ \text{df} = 9 \]
\[ \chi_{\alpha/2}^2 = \chi_{.025}^2 = 19.023 \]
\[ \chi_{1-\alpha/2}^2 = \chi_{.975}^2 = 2.7 \]

**DRAW A GENERIC \( \chi^2 \) graph with critical regions.**

Another example:

EX// Find the critical values of the \( \chi^2 \) for a 90% confidence interval with sample size of 15.

C=.90 \( \Rightarrow \) \( \alpha = .10 \) \( \Rightarrow \) \( \alpha/2 = .05 \) \( \Rightarrow \) \( 1-\alpha/2 = .95 \)

\[ \text{df} = 14 \]
\[ \chi_{\alpha/2}^2 = \chi_{.05}^2 = 23.685 \]
\[ \chi_{1-\alpha/2}^2 = \chi_{.95}^2 = 6.571 \]

**DRAW A GENERIC \( \chi^2 \) graph with critical regions.**

EX// #18 page 353
Quarters are currently minted with weights having a mean of 5.67 g and a standard deviation of 0.062 g. New equipment is being tested. A sample of 24 is obtained from the new equipment and it has a standard deviation of 0.049. Construct a 95% confidence interval of \( \sigma \).

\[ s = 0.049 \quad n = 24 \]
\[ \text{df} = 23 \]
\[ c = .95 \Rightarrow \alpha = .05 \Rightarrow \alpha/2 = .025 \Rightarrow 1-\alpha/2 = .975 \]
\[ \chi_{\alpha/2}^2 = \chi_{.025}^2 = 38.076 \]
\[ \chi_{1-\alpha/2}^2 = \chi_{.975}^2 = 11.689 \]

EX// #21 page 353
Below is a sample of credit scores for car loans. Use the data to construct a 99% confidence interval for the standard deviation for credit scores for all applicants for car loans.

**USE STATCRUNCH TO FIND \( S \)**

\( \left( \sqrt{15*95.89767^2} \right) = \left( \sqrt{32.801} \right) \)
\( \left( \sqrt{15*95.89767^2} \right) = \left( \sqrt{4.601} \right) \)

\( \left( \sqrt{4205.526} \right) = \left( \sqrt{29981.623} \right) = (64.85, 173.15) \)
s = 95.89767
n=16
df=15
c=.99⇒ α = .01⇒ α\2 = .005⇒ 1-α\2 = .995
\chi^2_{\alpha/2} = \chi^2_{.005} = 32.810
\chi^2_{1-\alpha/2} = \chi^2_{.995} = 4.601

GROUP WORK page 353 #19
(.54, .77)

*This is also a good place to do a review about how to calculate standard deviation.

page105 #5 s^2 = 89.6, s = 9.5
#3 page 146 Sullivan

CHAPTER 7 SUMMARY POWER POINT#7

Confidence Interval for μ with known σ:
\[ \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \]

Find sample size: \[ n = \left( \frac{z_{\alpha/2} \sigma}{m} \right)^2 \]

Confidence Interval for μ with unknown σ:
\[ \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \]

Confidence Interval for p:
\[ \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \]

Finding the sample size:
\[ n = \frac{z^2_{\alpha/2} \hat{p}\hat{q}}{E^2} \quad \text{OR} \quad n = \frac{z^2_{\alpha/2}}{4E^2} \]

Confidence Interval for σ:
\[ \left[ \frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \right] = \left[ \sqrt{\frac{(n-1)s^2}{\chi^2_{R}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{L}}} \right] \]