

These problems are to be done individually; no consultation with others is permitted. Write in the space provided. Exact answers are required unless otherwise stated. Show all necessary work in an orderly manner. Use other side if necessary. Place numerical answers in the appropriate blanks. All problems have equal weight. Write clearly and neatly. Explanations must correctly apply calculus ideas from class.

1. Evaluate $\int 8x^3 \cdot \sqrt[3]{3x^4 + 4} dx$

$$\frac{1}{2} (3x^4 + 4)^{4/3} + C$$

$$u = 3x^4 + 4$$

$$du = 12x^3 dx$$

$$= \int 8x^3 \sqrt[3]{u} \frac{du}{12x^3}$$

$$= \frac{2}{3} \int \sqrt[3]{u} du = \frac{2}{3} \int u^{1/3} du = \frac{2}{3} \cdot \frac{3}{4} u^{4/3} + C = \frac{1}{2} (3x^4 + 4)^{4/3} + C$$

2. Evaluate: $\int \frac{4x^3}{\sqrt{4-x^2}} dx$

$$-2 \left[8\sqrt{4-x^2} - \frac{2}{3}(4-x^2)^{3/2} \right] + C$$

$$u = 4 - x^2 \quad x^2 = 4 - u$$

$$du = -2x dx$$

$$dx = -\frac{du}{2x}$$

$$\int \frac{4x^3}{\sqrt{u}} \left(-\frac{du}{2x}\right)$$

$$-\frac{4}{2} \int \frac{x^2}{\sqrt{u}} du$$

$$-2 \int \frac{4-u}{u^{1/2}} du$$

$$-2 \int (4u^{-1/2} - u^{1/2}) du$$

$$-2 \left[8u^{1/2} - \frac{2}{3}u^{3/2} \right] + C$$

3. Evaluate $\int_{\pi/8}^{3\pi/8} \sin^6 4x \cdot \cos 4x dx$ by making a substitution and changing the limits on the integral. Show details of the change.

$$u = \sin 4x$$

$$du = \cos 4x \cdot 4 dx$$

$$\frac{1}{4} du = \cos 4x dx$$

$$x = \frac{\pi}{8} \quad u = \sin \frac{4\pi}{8} = 1$$

$$x = \frac{3\pi}{8} \quad u = \sin \frac{12\pi}{8} = -1$$

$$\frac{-1/4}{1/4}$$

$$\int = \int_1^{-1} u^6 \cdot \frac{1}{4} du = \frac{1}{4} \int_1^{-1} u^6 du = \left. \frac{1}{28} u^7 \right|_1^{-1}$$

$$= -\frac{1}{28} - \left[\frac{1}{28} \right]$$

$$= -\frac{2}{28}$$