1. WHY STUDY THIS STUFF?

It is common for a college student to stand in awe of the professor and perhaps to regard a textbook writer as a person with some kind of superhuman knowledge. What is often overlooked is the fact that we professors were once students ourselves and many of us thought that we would never attain to the lofty heights of understanding that our teachers displayed so casually during the course of their lectures. Many of us were right! Let it be known right now that a substantial number of readers of this book are much brighter and faster than I ever was. If I have an advantage over them, it is merely due to experience, not to innate ability. My purpose in writing this book is simply to share some things that I have learned over the years; ideas and techniques I have found helpful and which I believe can help students to progress faster and further along the road to deeper mathematical understanding.

There are students who have done quite well in mathematics up to and including calculus but who find that their first encounter with upper division math is a somewhat traumatic experience. I know this very well. I was one of them. After getting A’s and B’s in all my math classes during high school and my first two years in college, I was shocked to get C’s and a D as a junior. In retrospect, I realize now that the main reason for my difficulties was the fact that I had excelled in the purely computational aspects of math, but I really was unprepared for the rigors of thinking things through very carefully and writing my analyses down in a way that would be recognizable to the professor as showing that I truly understood what was going on. In short, I was terrible at proving theorems, and that was the main focus of all the upper division courses I took.

After learning the hard way, I eventually became a professor myself, and have spent a substantial part of my career teaching classes for those who intend to be secondary school teachers. My experience tells me that the gap between elementary and advanced math is, for most persons, more of a yawning chasm than a gap. The problem seems to be that very few really understand the mechanics and processes that constitute a correct mathematical proof. To progress towards mathematical maturity, it is necessary to be trained in two aspects: the ability to read a proof and the ability to write a proof.

I might very well have called this book “Mathematical Ideas I wish I had known Much Earlier in My Career”. The emphasis, as you may have guessed, is on proof. But I intend to do more than simply prove a bunch of theorems. All mathematically literate persons should be conversant with the basics of math, including a knowledge of logic, sets, and the different kinds of numbers and their properties. So, I intend to cover the ground of elementary mathematics, but with special attention paid to its logical structure and especially with an emphasis on how the theorems are proved.

Many, if not most, of the individual facts presented in this book will be known to you. This is deliberate. By dealing with material that you have already learned, you will be in a better position to concentrate on the underlying thought processes and practice in a variety of applications the many theorem-proving techniques that should be part of every mathematician’s arsenal.

If you are now suffering from that cold, clammy, uptight feeling that often accompanies a mention of that dreaded word “proof”, rest assured that with some effort (maybe lots of effort!), it is possible to reach that wonderful place in which you at least have an idea of where to go and what to do when someone says: Prove.