All statements are to be proved within the context of our axiomatic theory. Be careful to organize your steps properly. If you don’t begin and end appropriately, you may prove something, but it may not be what you are asked to prove. So beware! You may use ONLY numbered theorems for your reasons (and, of course, logic), not exercises or things we may have mentioned in class (even if true).

1. Prove algebraically: \((A \cup B) - (C - A) = A \cup (B - C)\)

2. Prove in your own inimitable way: \(G - (H - G) = G\)

3. \(G \subseteq H \land K \subset L \implies G - L \subset H - K\).

4. If \(R\) is a transitive relation, then \(R^{-1}\) is also a transitive relation.

5. If \(R, S\) and \(T\) are relations and \(R \subseteq S\), then \(R; T \subseteq S; T\).

6. Suppose \(R\) and \(S\) are relations. Of the two sets \(\text{dom}(R - S)\) and \(\text{dom} R - \text{dom} S\), one is a subset of the other. Decide which, prove it, and give a counterexample for the reverse inequality.

\[\begin{align*}
\text{1. } (A \cup B) \cap (C \cap \neg A) &= (A \cup B) \cap (\neg C \cup A) = A \cup (B \cap \neg C) = A \cup (B - C) \\
\text{2. } G \cap (H \cap \neg G) &= G \cap (\neg H \cup G) = G & \text{Th 5.1}
\end{align*}\]

7. Suppose \(x \in E\). Then \(x \in E\) and \(x \notin L\). \(\therefore x \notin H\). If \(x \in K\), then \(x \notin L\), which is a contradiction. \(\therefore x \notin H - K\).

8. Suppose \(\langle x, y \rangle \in R^{-1}\) and \(\langle y, z \rangle \in R\). Then \(\langle y, x \rangle \in R\) and \(\langle z, y \rangle \in R\).

Since \(R\) is transitive, \(\langle z, x \rangle \in R\).

9. Suppose \(\langle x, y \rangle \in R; T\). There is \(z\) such that \(\langle x, z \rangle \in E\) and \(\langle z, y \rangle \in T\). By hyp \(\langle x, z \rangle \in E\).

10. Suppose \(x \in \text{dom} R - \text{dom} S\). There is \(y\) such that \(\langle x, y \rangle \in R\). If \(\langle x, y \rangle \in S\), then \(x \in \text{dom} S\). Contrad.

\[\begin{align*}
\text{CE. Let } R &= \langle 1, 2 \rangle \\
\text{dom}(R - S) &= \{1\} \\
\text{dom} R - \text{dom} S &= \emptyset
\end{align*}\]