Differential Equations

Quiz 2

Name: Answer Key

1. (2 pts) Consider the initial value problem: \((x - 4)y' + 2y = x\), \(y(1) = 1\). 
State the largest interval for which this initial value problem has a unique solution.

Standard form: \(y' + \frac{2}{x-4} \frac{y}{x-4} = \frac{x}{x-4}\) 

\(P(x) \& f(x)\) must be continuous.

\(P(x) = \frac{2}{x-4}\) continuous for all \(x \neq 4\)

\(f(x) = \frac{x}{x-4}\) so interval can be \((-\infty, 4)\) or \((4, \infty)\)

So interval is: \((-\infty, 4)\)

2. \(y_1(x) = x^2\) and \(y_2(x) = x^4\) are two solutions to \(x^2y'' - 5xy' + 8y = 0\) on the interval \((0, \infty)\).

(3 pts) (a) Determine whether \(\{y_1, y_2\}\) is a linearly independent set of functions on \((0, \infty)\) by computing the Wronskian of \(y_1\) and \(y_2\).

(2 pts) (b) Explain why \(\{y_1, y_2\}\) is a fundamental set of solutions on this ODE on \((0, \infty)\).

(3 pts) (c) Write the general solution to the ODE on \((0, \infty)\).

\[a) \quad W(y_1, y_2) = \begin{vmatrix} x^2 & x^4 \\ 2x & 4x^3 \end{vmatrix} = 4x^5 - 2x^5 = 2x^5 > 0 \text{ for all } x \in (0, \infty).\]

Since \(W(y_1, y_2) \neq 0\) for all \(x \in (0, \infty)\), it follows that \(\{y_1, y_2\}\) is linearly independent on \((0, \infty)\).

b) Since \(\{y_1, y_2\}\) is a set of 2 linearly independent solutions to the 2nd order linear homogeneous ODE on \((0, \infty)\), it is a fundamental set of solutions to the ODE on \((0, \infty)\).

c) \(y(x) = c_1x^2 + c_2x^4\)