1. In the first exam, you are responsible from sections 1.1, 1.2, 2.1, 2.2, 2.3, 3.1, 4.1, 4.2. You can use only scientific calculators (that is calculators without the ability to graph) during exam.

2. Consider the differential equation: \( y' = \frac{\sqrt{y}}{x} \). State whether this equation is 
   - linear or nonlinear in \( y \) (Explain)
   - separable or not separable (Explain)
   - autonomous or not autonomous (Explain)

3. Given: \( \frac{dy}{dx} = \frac{\sqrt{y-1}}{x^2} \).
   Solve by separation of variables. Write the solution in explicit form.
   Specify any singular solutions that are apparent from the solution process. (That is, determine if any lost solutions are singular solutions.)

4. Solve the IVP: \( x^2y' + xy = 1, y(1) = 2 \). Write the solution in explicit form. Give the largest interval \( I \) over which this solution is defined.

5. Suppose that a large mixing tank initially holds 200 gallons of water in which 50 pounds of salt have been dissolved. Pure water is pumped into the tank at a rate of 3 gal/min, and when the solution is well stirred, it is then pumped out at the same rate. Determine a differential equation for the amount \( A(t) \) of salt in the tank at time \( t \). What is \( A(0) \)?

6. Consider the autonomous differential equation: \( \frac{dy}{dx} = 1 - y^2 \).
   (a) Find all critical points and the phase portrait.
   (b) Classify each critical point as asymptotically stable, unstable, or semi-stable.
   (c) State the equilibrium solutions.
   (d) Sketch the equilibrium solutions and typical solution curves in the region of the \( xy \)-plane determined by the graphs of the equilibrium solutions.

7. Consider the homogeneous differential equation: \( x^2y'' + 5xy' - 12y = 0 \).
   Given that \( y_1 = x^2 \) is one solution to this homogeneous equation, find the general solution using reduction of order.

8. \( y_1(x) = e^x \) and \( y_2(x) = e^{-3x} \) are two solutions to \( y'' + 2y' - 3y = 0 \) on the interval \((-\infty, \infty)\).
   (a) Determine whether \( \{y_1, y_2\} \) is a linearly independent set of functions on \((-\infty, \infty)\) by computing the Wronskian of \( y_1 \) and \( y_2 \).
   (b) Explain why \( \{y_1, y_2\} \) is a fundamental set of solutions on this ODE on \((-\infty, \infty)\).
   (c) Write the general solution to the ODE on \((-\infty, \infty)\).