Abstract

This study focuses on fluid mechanics approaches and Herschel-Bulkley constitutive equation to develop a theoretical model for predicting the behavior of field-controllable electro-rheological (ER) and magneto-rheological (MR) fluid dampers. The aim of this research is to provide an accurate theoretical model for analysis, design, and development of control algorithms for ER/MR dampers. The Herschel-Bulkley quasi-steady analysis is extended to include the effect of fluid compressibility. The major advantage of the proposed model is its dependency only on the geometric and material properties of the device. The theoretical results are validated by an experimental study. It is demonstrated that the proposed fluid mechanic-based dynamic model can effectively predict the behavior of field-controllable fluid dampers.

1. Theoretical Modeling

The derived theoretical model is applied to a new prototype MR fluid damper. The schematic of this damper is shown in Figure 1. The damper has a through-rod design, meaning that the ratio of fluid volume to rod volume in the damper is constant over a stroke. The controllable MR valves are arranged in the piston. An enlarged cross-sectional view of the MR fluid damper’s piston/rod assembly is shown in Figure 2. The fluid flow path through the piston and the geometric dimensions used in the theoretical formulation are shown in Figure 2. The general mass flow rate continuity for a fluid volume is

\[ \left[ \frac{\partial q_x}{\partial t} + \nabla \cdot \mathbf{q} \right] = 0 \]

Equation (3) represents a general mass flow rate continuity equation accounting for the fluid compressibility. For the specific MR fluid damper, \( q_x \) is zero, and the flow continuity Eq. (2) for chambers one and two, respectively, one obtains:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]  

where \( \rho \) is the density in the mass, \( \rho \) is the input flow rate, and \( q_x \) is output flow rate. Considering the compressibility of the fluid, one has

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = -\nabla p + \rho \frac{\partial \mathbf{u}}{\partial t} \]

Equation (2) is a general mass flow rate continuity equation that is written to include the effect of fluid compressibility. For the specific MR fluid damper, \( \rho \) and \( \mathbf{u} \) are a constant fluid density, and

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = -\nabla p + \rho \frac{\partial \mathbf{u}}{\partial t} \]

Here the piston displacement, \( \Delta p \), is the effective area of the piston, and \( \rho \) and \( \mathbf{u} \) are the effective bulk modulus of the fluid in chambers one and two, respectively. At room temperature, knowing the volume of air present per unit volume of oil and assuming a perfectly rigid container, the effective bulk modulus is estimated as

\[ \frac{1}{\rho} = \frac{1}{\rho} \Delta p \]

Therefore, it is evident that air content in the fluid can drastically reduce the effective bulk modulus. In practical applications, air is always present in the system, and the effectiveness of the design can significantly be affected, if the design process neglects the variation of compressibility. However, in this case, it is reasonable to assume that the effective bulk modulus \( \Delta p \) and \( \rho \) are the same constant. Assuming \( \Delta p = p + \alpha \Delta p \), and defining \( \gamma \) as

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = -\nabla p + \rho \frac{\partial \mathbf{u}}{\partial t} \]

where \( \gamma = \frac{\rho}{\Delta p} \) is the piston velocity.

According to the flow paths of fluid through the piston shown in Figure 2, the auxiliary pressure drop \( \Delta p_2 \), between chamber one and chamber two is mainly related to the pressure drop \( \Delta p_2 \) due to the fluid velocity in the central channel, and the pressure drop \( \Delta p_{22} \) across the MR valve normal to the axis of the shafts. When the MR fluid is activated, the viscous pressure drop \( \Delta p_{22} \) is much smaller than pressure drop \( \Delta p_2 \) in the MR valves. Therefore, the pressure drop in the central path is neglected. Thus, the total volume flow rate through MR valves can be expressed as the function of pressure drop, \( \Delta p_2 \)

\[ \frac{\partial q_x}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = -\nabla p + \rho \frac{\partial \mathbf{u}}{\partial t} \]

Therefore, the total pressure drop across the piston can be expressed as

\[ \Delta p_2 = (\gamma - 1) \frac{\partial q_x}{\partial t} + \nabla \cdot (\rho \mathbf{u}) \]

For the given MR fluid damper geometry, and assuming incompressible fluid in the piston channel, the total volume flow rate is

\[ \frac{\partial q_x}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = -\nabla p + \rho \frac{\partial \mathbf{u}}{\partial t} \]

where \( \gamma \) is the number of MR valves. Using Equation (7), (8) and (9), the pressure drop \( \Delta p_2 \) can be obtained. The pressure drop \( \Delta p_2 \) contributes a viscous damping force to the overall damping force, therefore, the total pressure drop across the piston can be expressed as

\[ \Delta p = (\gamma - 1) \frac{\partial q_x}{\partial t} + \nabla \cdot (\rho \mathbf{u}) \]

where \( \Delta p \) is the damping force. For a Newtonian Poiseuille flow in a circle, the pressure drop \( \Delta p_2 \) is

\[ \Delta p_2 = \frac{10 \pi \eta \Delta r_2 / \Delta r}{10000 \pi \Delta r_2 / \Delta r} \]

where \( \Delta r_2 \) is the bulk modulus of the pure oil and is in atmospheres. \( \eta \) is the viscosity of the oil in centistokes (cSt). \( \rho \) is the density of the fluid in kg/m³. \( \Delta r_2 \) is the ratio of a volume of air dispersed in oil (from 0.01% to 1%), and \( \Delta p \) is the pressure drop. In the MR fluid damper considered here, the chambers mean value pressure is about 5.5Mpa. If 1% of air is entrapped in the fluid, the effective bulk modulus will be \( 4.5 \times 10^6 \)Pa, while the effective bulk modulus is \( 4.5 \times 10^6 \)Pa at 0.01% or air content.

3. Validation of the Proposed Model

A prototype MR fluid damper shown in Figure (1) is constructed and tested on an Instron 4465 hydraulic dynamometer. The fluid has a maximum stress of 12.24 cm (5.6 inches), and it is equipped with a 22 KN/2500 lbf load cell that measures the damping forces of the MR fluid damper. A displacement/velocity transducer measures the displacement and velocity of the hydraulic actuators. Experimental data is collected at a sampling rate of 200Hz.

A series of tests is conducted to measure the response of the damper under various sinusoidal loading conditions. A performance test consists of frequency sweep held at constant displacement input. Each specific amplitude and frequency represents a run of the performance test. In each test, the input electric current applied to the prototype MR fluid damper is kept at a constant level of 0.1, 0.5, 1.5, and 2.0 Amps. The geometric and material properties are given in Table 1. The yield stress \( \sigma_y \) is a function of the applied magnetic field strength.

\[ \sigma_y = \frac{1}{10000 \pi \Delta r_2 / \Delta r} \]

The yield stress \( \sigma_y \) is in atmospheres, and the input current \( I \) for an MR fluid damper is: \( \sigma_y = \frac{1}{10000 \pi \Delta r_2 / \Delta r} \)

4. Conclusions

A dynamic model is presented based on fluid mechanics and the Herschel-Bulkley constitutive equation to predict the behavior of ER/MR fluid dampers. The effect of fluid compressibility within the damper is considered in the derivation of the effective bulk modulus in the proposed model. The theoretical model presented is validated by comparing the analytical results with experimental data for a prototype MR fluid damper. It is demonstrated that the proposed fluid mechanics-based model can accurately predict the dynamic response of an MR damper over a wide range of operating conditions.

5. Acknowledgements

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Table 1. Material and Geometric Properties Used in the Fluid Mechanics Based Model.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
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<td>( \rho )</td>
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</tr>
<tr>
<td>( \eta )</td>
<td>1 cSt</td>
</tr>
<tr>
<td>( \Delta r_2 )</td>
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<tr>
<td>( \Delta r_2 )</td>
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Figure 1. Schematic of the prototype MR fluid damper.

Figure 2. Comparison between the proposed model and experimental results for a sinusoidal motion at 1.0 Hz and 1.016 cm (0.40in) amplitude at 1.0 Amps electric current input. The effective bulk modulus is \( \sigma_y = \frac{1}{10000 \pi \Delta r_2 / \Delta r} \).