LYAPUNOV-BASED CONTROL OF A BRIDGE USING MAGNETO-RHEOLOGICAL FLUID DAMPERS

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Abstract

The study focuses on the effect of semi-active magneto-rheological fluid (MRF) dampers in reducing the response of a scaled bridge structure subjected to a random load. A fluid-mechanics-based model that can characterize the nonlinear dynamic behavior of MRF dampers is employed. A state-variable model for an integrated system of a 1/12-scaled, two-span bridge and two MRF dampers is established. A feedback control law is employed based on Lyapunov approach that guarantees the system stability for uncertain boundary input disturbances. An output feedback Lyapunov controller is implemented in the experimental study to verify the presented method. Both the theoretical and experimental studies show that the Lyapunov-based control systems can effectively reduce the relative displacement between the deck and the abutment of the bridge when subjected to various input motions. In addition, when compared to experimental results, it is demonstrated that the proposed analytical model can predict the response of the system accurately.

Introduction

Semi-active control of bridges under random loading such as earthquakes, strong storms, and vehicular traffic has received attention in recent years [1]. In order to achieve active and passive systems, semi-active control can offer the combined advantages of both systems with its control flexibility and low energy requirement. A variety of semi-active control devices have been developed for the purposes of structural vibration control. Examples of such devices include variable orifice fluid dampers, variable stiffness devices, vibration control isolators, and controllable MRI fluid dampers.

This paper aims at the vibration control of a scaled bridge model for a proof-of-concept evaluation utilizing MRF dampers as the semi-active control device. The structure considered in this study is a two-span scaled bridge. The MRF dampers are developed at the University of Nevada, Reno (UNR). A bang-bang control strategy based on Lyapunov stability theory is proposed for controlling MRF dampers to reduce bridge response due to random vibrations. A theoretical study incorporating a newly developed model that can accurately capture the dynamic behavior of MRF dampers, is proposed.

Theoretical Modeling

The bridge system can be modeled as a single-degree-of-freedom system. The equation of motion subjected to a ground excitation can be expressed as:

\[ M \ddot{u} + C \dot{u} + K u = F(u) + Fext \]  \hspace{1cm} (1)

where \( M \) is the mass of the bridge deck, \( C \) is the structural damping coefficient, \( K \) is the column stiffness, \( F(u) \) are relative displacement, velocity and acceleration of deck with respect to ground, respectively, \( F \) is the input acceleration, and \( F_{ext} \) is the control damping force of the MRF dampers. A dynamic model based on fluid mechanics approach utilizing a Herschel-Bulkley constitutive equation has been demonstrated to accurately predict the dynamic responses of the UNR MRF-001 damper over a wide range of operating conditions [2]. The model includes dissipative as well as stored energies. Combining the dynamic modeling of the MRF dampers with the bridge plastic equation, the model for a controllable bridge system can be expressed as:

\[ M \ddot{u} + C \dot{u} + K u = F(u) + F_{cont}(\dot{u}) \]  \hspace{1cm} (2)

where the rate of pressure drop across the piston can be expressed as follows:

\[ \dot{u} = \dot{C} \frac{1}{\rho} \left( \frac{dP}{dt} - \frac{1}{2} \rho \dot{u} \dot{u} \right) \]  \hspace{1cm} (3)

where

\[ \dot{C} = \frac{32 \mu}{3 L D^2} \]  \hspace{1cm} (4)

Here, \( \dot{C} \) is the pressure drop between damper chamber one and chamber two, \( \rho \) is the density of the fluid in the chambers, \( \mu \) is the dynamic viscosity of the fluid, \( D \) is the diameter of the fluid tube, \( L \) is the length of the fluid tube, and \( \dot{P} \) is the rate of pressure drop across the piston.

Controller Design

In the study the Lyapunov direct method is used to design the controller. Consider a Lyapunov function candidate, as follows:

\[ V = \frac{1}{2} \int_{0}^{t} \left( \dot{u}^2 + F_{cont}^2 \right) dt \]  \hspace{1cm} (5)

where \( \dot{u} \) is the positive definite solution of the algebraic Lyapunov equation:

\[ (M - \alpha \rho I) \dot{u} + C \dot{u} + K u = 0 \]  \hspace{1cm} (6)

\[ \alpha \] is a positive constant.

The Lyapunov control input can be expressed as:

\[ u = -\frac{1}{2} \dot{C} \left( 2 \rho \dot{u} - \dot{u} \dot{u} \right) \]  \hspace{1cm} (7)

The minimum value of \( u \) can be obtained by

\[ \dot{V} = -\frac{1}{2} \int_{0}^{t} \dot{C} \left( 2 \rho \dot{u} - \dot{u} \dot{u} \right)^2 dt \]  \hspace{1cm} (8)

In order to minimize \( \dot{V} \) for all possible \( \dot{u} \), one needs to only minimize \( \dot{u}^2 \) for \( \dot{u} \| \dot{u} \| \) at \( \| \dot{u} \| = \alpha \) if \( \dot{u} \) is a constant.

Thus, \( \dot{u} = \frac{\| \dot{u} \|}{\alpha} \) and

\[ \dot{C} = \frac{32 \mu}{3 L D^2} \frac{\rho \dot{u} \| \dot{u} \|}{\alpha} \]  \hspace{1cm} (9)

Since \( \| \dot{u} \| = \dot{C} \frac{1}{\rho} \left( \frac{dP}{dt} - \frac{1}{2} \rho \dot{u} \dot{u} \right) \)

\[ \dot{C} \frac{1}{\rho} \left( \frac{dP}{dt} - \frac{1}{2} \rho \dot{u} \dot{u} \right) = \frac{\| \dot{u} \|}{\alpha} \]  \hspace{1cm} (10)

Thus, a control scheme is obtained, as follows:

\[ \frac{\| \dot{u} \|}{\alpha} = \frac{\rho \dot{u} \| \dot{u} \|}{\dot{C} \frac{1}{\rho} \left( \frac{dP}{dt} - \frac{1}{2} \rho \dot{u} \dot{u} \right)} \]  \hspace{1cm} (11)

The control law requires measurements of relative displacement, relative velocity and internal pressure drop. A block diagram of the Lyapunov based semi-active control system is shown in Figure 2.

Conclusion

This study was focused on the application of semi-active MRF dampers for controlling the response of a bridge structure system subjected to random vibrations. Experimental study was performed on an integrated system consisting of both 1/12 scaled, two-axle bridge and two MRF dampers. Theoretical modeling of the damper was performed utilizing a fluid-mechanical model that includes the nonlinear dynamic behavior of MRF dampers. A feedback bang-bang control law was developed based on Lyapunov approach to stabilize the coupled structure/damper system. Both the theoretical and experimental studies show that the Lyapunov-based control system can effectively reduce the bridge relative displacement between the deck and the abutment when subjected to various input motions.

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References


