Gas Transport During Oscillatory Flow in a Network of Branching Tubes

A transport coefficient was measured for a range of oscillatory flow conditions in a branching network of tubes. Measurements were made both across the first generation of a three-generation network and the second generation of a four-generation network. The results for these two series of tests were similar, indicating that there was no significant effect due to the system boundaries. The results are cast in terms of an effective axial diffusion coefficient of the form

$$D_{\text{eff}} = (\pi + 0.50V_1^{1.86}f^{0.51}) \text{cm}^2/\text{s}$$

where $\pi$ is the molecular diffusivity, $V_1$ is the local stroke volume (cc), and $f$ is the oscillation frequency (Hz). These results are compared to those obtained by other investigators in branching systems of similar geometry. At low frequency, this result is found to be in approximate agreement with the steady flow result of Scherer, et al. [15]. This expression differs from the oscillatory flow results of Tarbell, et al. [19] for liquids, primarily in terms of the effects of oscillation frequency.

Introduction

These experiments were motivated by the need for a better characterization and understanding of the transport mechanisms which produce gas exchange in High Frequency Ventilation (HFV). Under conditions of ventilation by high frequency (5–15 Hz) and low tidal volume (25–100 mL), it has been shown that normal rates of gas exchange can be achieved both in test animals and in humans. Because the tidal volumes used in HFV are generally smaller than the pulmonary deadspace volume, processes other than direct convection to the respiratory zone must be present.

Several attempts have been made to simulate the exchange of gases through the respiratory system due to transport of a diffusive nature (Fredberg [6], Slutsky, et al. [17]) and due to direct alveolar ventilation (Kamm, et al. [11]). Of these, the diffusive transport models are most appropriate for very small tidal volumes, whereas the model of direct alveolar ventilation becomes increasingly valid as tidal volumes become comparable to and exceed the physiologic deadspace. The first diffusive transport models treated the lung as a series of resistances to gas transport, with the resistance of each segment (typically a single generation) being determined by local flow conditions.

While these models were successful in predicting some of the basic features of the results from animal tests, they were acknowledged to be deficient in that they are based merely on reasonable estimates of the actual rate of dispersion within the lung rather than analyses or experiments in which the conditions of HFV were appropriately modeled. These estimates were derived from various sources, including experiments in steady flow through a branching model (Scherer, et al. [15]), steady turbulent pipe flow (Taylor [20]), and laminar oscillatory flow in pipes (Watson [22], Joshi, et al. [9]), none of which are directly applicable to the conditions of HFV in the lung without invoking assumptions of questionable validity.

Several groups have recently conducted experiments to help elucidate and quantify the mechanisms of transport under conditions of small-volume, high-frequency oscillation. Tarbell, et al. [19] and Azhar and Tarbell [2] have measured the dispersion of a liquid dye in a five-generation model. In Tarbell's experiments, oscillations are imposed until the bolus spreads over the entire network. The fluid is then allowed to drain and the distribution of dye is measured as a function of the volume of fluid expelled. Measurements of effective axial diffusivity are determined from the volume variance of the concentration distribution and therefore represent an overall network dispersion coefficient.

Some preliminary results have recently been reported for oscillatory gas flow in bifurcating systems of similar geometry (Ulman, et al. [21], Kamm, et al. [10]). While these experiments for gases and liquids all measure the rate at which axial dispersion occurs in branching tubes, there exist some important differences due to the possible influence of molecular diffusion. The results reported thus far, however, do not permit an assessment of the influence of molecular diffusion.

Two primary mechanisms have been proposed to account for gas exchange observed in HFV: convective streaming...
Fig. 1: Branching network used in the experiments. Shown is the three-generation network. The four-generation network contains one additional generation with 16 pistons.

Haselton and Scherer [8]) resulting from a net bi-directional flow when the velocity profile is averaged over an integer number of cycles, and augmented dispersion (Kamm, et al. [11]) resulting from a combination of a nonuniform axial velocity profile and some form of lateral or cross-stream mixing, producing axial transport which is diffusive in character.

Augmented dispersion exists in a variety of forms, depending on the nature of lateral transport. The cross-stream mixing can be due to molecular diffusion, or it can result entirely from convective motions as in turbulence or flows with significant velocities in the plane perpendicular to the tube axis. In the latter case, it becomes difficult to distinguish augmented dispersion from streaming. In the present experiments, both streaming and augmented dispersion are likely to be important.

The primary objective of these experiments is to obtain correlations which can be used in simulations of gas exchange in the lung by HFV. As described earlier, these results are generally applicable to airways from generation 5 to 13 of the Weibel [23] morphometric model of the human lung. In addition, however, these results in combination with those of other investigators, help us to better characterize the flow and gas transport processes and to address questions relating to the physical mechanisms affecting gas transport.

Experimental Methods

Rationale. The objective of these tests was to determine the dispersion coefficient for transport during conditions simulating HFV. For this purpose, we set up a steady condition in which a trace gas is infused at a constant rate at the "alveolar" end of the network. Measurements are taken only after a quasi-steady state has been reached throughout the network. This state is achieved once the cycle-averaged concentration at each measurement location is steady, which, by conservation of mass, indicates that the cycle-averaged mass flow rate of tracer through each generation is uniform. This not only simulates the conditions of HFV under steady-state conditions (O₂ and CO₂ being exchanged at constant rates), but also increases the accuracy of our experiments since the measurement can be repeated over many oscillation cycles and averaged to eliminate extraneous noise.

Apparatus. The model used in the experiments consists of a system of rigid, constant diameter tubes, constructed from 1-cm i.d. nalgene Y connectors as shown in Fig. 1. Each bifurcation has a branching angle of 60 deg and each tube segment has a length-to-diameter ratio of 3.5. Flow oscillations produced by eight (sixteen in the four-generation experiments) synchronously driven pistons and a constant infusion of pure tracer gas were both introduced at the "alveolar" end of the model so as to eliminate any uncertainties concerning the distribution of flow to each branch. Studies (Snyder, et al. [18]) in a model geometrically similar to ours have indicated that the flow distribution among lateral and medial branches varies with flow rate and can be quite sensitive to small geometrical asymmetries.

Measurement Methods. The tracer concentration was measured simultaneously at two sites using an infra-red (IR) absorption technique. The attenuation of monochromatic radiation by a gas with absorptivity of $\beta$ is given by Beer's law

$$I = I_o \exp(-\beta \bar{C} \Delta x)$$

(1)

where $I$ is the attenuated intensity of radiation, $I_o$ is the unattenuated intensity, and $\Delta x$ is the average volume fraction of the absorbing gas along a beam path of length $\Delta x$. The experiments were conducted using a helium-neon laser operating at 3391 μm (Jodon Model HN-5) and methane as the absorbing tracer gas due to the strong coincidence of the methane absorption band ($\beta = 9.4$ cm⁻¹ atm⁻¹) and the laser wavelength.

The laser beam was chopped, then split into three beams. One beam was directed toward a PbSe infrared detector while the other two were passed through glass coverslips mounted in the branching network at one pair of the locations identified in Fig. 1.

At measurement location (0), the pathlength averaged concentration can be written as

$$\bar{C} = [k_1 - \ln(e/e_0)] / 2a\beta$$

(2)
where $e_r$ is the output voltage of the reference detector, $e_i$ is the output voltage of the optical detector at measurement location ($i$), and $(k_1)$ is a constant which depends upon the alignment of the optical system.

We assume that the pathlength and cross-sectional averages are nearly equal to one another. This assumption breaks down in situations in which the cross-sectional variation in concentration is large. Even in these situations, however, we might expect the error at any two of the measurement locations identified in Fig. 1 to be in the same sense in which the errors would tend to cancel when calculating the local concentration gradient.

**Data Collection and Processing.** The detector output signals were passed through an RMS to DC converter (eliminating the chopping frequency) and then digitally processed (using a DEC MINC-1 Data Acquisition System). Additional signals from a piston position transducer and phase indicator were also digitized and processed. The phase indicator initiates the data acquisition sequence causing the A/D converters to sequentially sample the input signals at a rate of 100 points per cycle over successive intervals 1.5 cycles long. This procedure was repeated from 20 to 100 times and an ensemble average was computed to minimize random measurement error. The detector signals were then used to compute instantaneous values of the tracer concentrations (equation (2)). The signal indicating piston position was used to compute the bulk oscillatory flow rate. Each of these signals, representing a time-varying profile over a 1.5 cycle period, was then stored in digital form.

Samples of typical concentration profiles at the various measurement locations are shown in Fig. 2. The subscripts refer to measurement locations identified in Fig. 1.

**Experimental Protocol.** To conduct an experiment, the oscillatory flow and tracer infusion were started simultaneously using a methane flow rate selected so as to achieve methane concentrations of from 1 to 15 percent at the measurement sites. Concentrations were monitored until a steady-state was reached in which the cycle-average concentrations no longer varied with time. At that point, measurements were simultaneously made at two locations. When the measurement was completed, the oscillations were stopped, the system was purged with fresh air and a reference measurement was taken corresponding to zero concentration.

**Data Analysis.** Computer analysis of the outputs from the IR detectors yields values of methane concentration as a function of time at the two measurement sites. In addition, the time-mean methane flow rate in each tube segment is known since a steady-state had been reached prior to measurement. Using measured values of the methane flow rate $d_n$, values for an effective diffusivity can be computed from the following expression (see Joshi, et al. [9] for the complete derivation):

$$D_{eff} = \frac{\phi_m (x_1 - x_2) \ln[(1 - C_1)/(1 - C_2)]}{\ln(1 - C_1)/(1 - C_2)}$$  

where subscripts 1 and 2 refer to different measurement sites (either 1 and 2 or 2 and 3 in Fig. 1). In writing this expression, we make the implicit assumption that the transport process is diffusive in character as has been done by others in the past (Scherer, et al. [13], Tarbell, et al. [19]). However, neither our measurements nor those of others, provide definitive evidence of the diffusive nature of the transport process. In fact, Haselton and Scherer [7] have demonstrated that the results of Scherer, et al. [13] can be explained on the basis of a convective rather than diffusive process. The values of $D_{eff}$ determined from equation (3) are fit, by a double least-squares method, to a correlation of the form

$$D_{eff} - \kappa = \tilde{q} V_f f^x$$  

Here $\kappa$ is the molecular diffusivity, $V_f$ the local tidal volume in the tube segment (i.e., total stroke volume divided by 2 for first generation and 4 for second generation) and $f$ is the oscillation frequency. While other forms might be more appropriate as, for example, that obtained by the theoretical analysis of Erdogan and Chatwin [5] for steady flow dispersion in curved tubes, the correlation obtained using equation (4) provided a satisfactory fit of the data. This form is consistent with that used by Tarbell, et al. [19] to reduce their experimental results except that, for their experiments in liquid, it was not necessary to include the effects of axial molecular diffusion.

**Results**

The purpose of these experiments was to study and quantify gas transport during HFV. The range of experimental parameters was therefore selected so as to overlap, as much as possible, the range of HFV conditions found previously (Rosing, et al. [14]) to maintain adequate gas exchange in human subjects. In dimensionless terms, the experiments span a range of Peclet number ($Pe = 2Ua/x$ where $U$ is the cross-sectional mean velocity amplitude, $a$ is the vessel radius, and $x$ is the molecular diffusivity) from 5 to 1100 and dimensionless frequency ($\beta = \omega L/2$) from 1 to 12. In dimensional terms, this corresponds to tidal volumes ranging from 2 to 40 cc and frequencies from 0.2 to 13 Hz. At a tidal volume of 50 cc and frequency of 15 Hz, this corresponds to generations 5 to 13 in a lung with the morphology given by Weiβ [23]. These results cannot be directly applied to the first generations both because of this constraint on Re and also because the turbulence generated by a tracheal catheter or glottal apperture is absent in our tests.

In the calculation of $D_{eff}$ using equation (3), the time-averaged concentration is used. Our measurement technique, however, had sufficient time resolution to discern time variations in concentration during the course of an oscillation cycle. Two typical traces of concentration versus time are shown in Fig. 2.

Two series of experiments were conducted that span roughly the same range of parameters. In the first (Series A), measurements were made across the first generation of the branching tree, at positions 1 and 2 (see Fig. 1). In the second (Series B) concentrations were measured across the second generation at 2 and 3. Since we anticipated differences in the two data sets, we performed the curve-fitting analysis on each set separately. In addition, we found that the results from Series A satisfied different relations depending on the range of dimensionless frequency. The correlations of all our data are presented in Table 1, including the single expression obtained when all the data from both generations were analysed together.

Shown also in Table 1 are values for a second set of constants corresponding to the dimensionless relationship (see dimensional analysis in the forthcoming)

$$D = \frac{D_{eff} - \kappa}{\kappa} = \tilde{q} Pe^\beta f^x$$  

While it is convenient to express our results in dimensionless form, the dependence on molecular diffusivity suggested by this equation has not been tested in our experiments since we used a single gas mixture. Consequently, the dimensionless transport coefficient given in the foregoing may also exhibit a dependence on Schmidt number not evident in our experiments.

The fit of our data to this particular form can be seen by plotting $(D_{eff} - \kappa)/(\kappa 1/2 \tilde{q} f^\beta)$ versus $Pe$. According to equation (5), this plot has a slope of $f$ on a log-log plot as shown in Fig. 3.

**Discussion**

**Effects of Model Geometry.** One objective of these ex-
periments was to determine expressions that can be used in model simulations of gas exchange during typical conditions of HFV. The extent to which these results are useful for this purpose depends upon the realism incorporated into the model. In our experiments several geometrical features of the model differ from those typically found in the lung. For example, the transition from one generation to the next is abrupt in the model in that the corners are sharp and the change in area (a factor of two at each branch) is larger than one typically finds in the airways. These differences are likely to promote flow separation, especially during inspiration. The velocity fluctuations and eddy motion associated with the separated region could influence the rate of gas exchange. These same factors could cause an earlier transition to turbulence in the model than in the lung, but since the peak values of Stokes layer Reynolds number (Re/α) in our tests were always less than 200 it seems unlikely that we entered the turbulent regime (Akhavan [1]). Furthermore, we found no evidence in our experimental results to suggest a transition to turbulence as observed by Azhar and Tarbell [2].

Boundary Effects. In experiments conducted in networks with relatively few generations, one must also consider the importance of end effects. We would expect, for example, that fluid near the ends would exhibit less mixing due to secondary flow since that fluid resides in a straight tube segment during a larger part of the cycle than fluid in the more central regions of the model. This effect would be most evident in measurements made at the first generation where the mean fluid displacement is greatest. We found, however, that the differences between the first and second generation (Series A and B) were relatively small. Although the curve-fitting procedure yields somewhat different results for the two test series (Table 1), when the two data sets are combined there is a relatively small decrease in the accuracy of the curve fit.

In analysing the data, we obtained the best fit to the data by dividing the results from Series A into two groups depending on whether β was > or < 4. However, we found no clear advantage when the Series B results were processed in the same way. While this seems to suggest that the transition to quasi-steady flow occurs at lower values of β, this may simply reflect the relatively large experimental error in these low-frequency measurements as well as the narrow range of β tested. To resolve these questions will require more extensive tests at yet lower values of β.

Dimensional Analysis of Unsteady Dispersion. To provide a framework for the consideration of the effects of unsteadiness and molecular diffusion to follow, we first cast the axial dispersion coefficient in nondimensional form. In dimensional terms, we express \( D_{\text{eff}} - \kappa \) (representing transport above and beyond that due to axial diffusion alone) as a function of six dimensional parameters

\[
D_{\text{eff}} = f(T, a, U, T, \nu, \kappa, g)
\]

where \( T \) is the period of oscillation \( (T = 2\pi/\omega) \), and \( g \) symbolically represents all the parameters associated with airway geometry. A dimensionless form of \( D_{\text{eff}} \) is

\[
(D_{\text{eff}} - \kappa)/U^2 = f(Sc, \beta, Pe, Sc, G)
\]

where \( Sc \) is the Schmidt number \( (Sc = \nu/\kappa) \) and \( G \) denotes all dimensionless parameters having to do exclusively with system geometry.

The dimensionless time in this expression is the square of the ratio formed by the radial diffusion time \( (\tau_{\text{rad}} = a^2/\kappa) \) and the cycle period \( (T) \). Therefore we can rewrite (7) as

\[
(D_{\text{eff}} - \kappa)/U^2 = f(Sc, \beta, Pe, Sc, G)
\]

When the theoretical results for laminar oscillatory flow dispersion in a straight tube (Watson [22]) are expressed in the form of equation (8), we obtain the curve plotted in Fig. 4. The comparison in Fig. 4 provides information on the importance of \( G \) in equation (8). Note that when \( \tau_{\text{rad}} \) alone is decreased the rate of dispersion will either decrease or increase depending on whether \( \tau_{\text{rad}} < \) or \( > T \), respectively.

An alternate, but entirely equivalent dimensionless form can be derived which is useful in comparing our results to those of other investigators.

\[
D = (D_{\text{eff}} - \kappa)/U^2 = f(Sc, \beta, Pe, Sc, G)
\]

Under certain circumstances, one or more of the dimensionless groups in equation (9) can be eliminated. Several

![Graph](image)

**Fig. 3** Comparison of all experimental data to the regression formula

\[
D = 0.078 Pe^{1.46} \beta^{-1.46}
\]

| Coefficients are provided for two general expressions: |
| \((D_{\text{eff}} - \kappa)/U = \bar{Q} Pe^{1.46} \beta^{-1.46} \) |
| and |
| \(D_{\text{eff}} = \kappa + q V f_1 \tau_1 s\) |

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(1) Computed from the expression \( \bar{Q} = D_{\text{eff}} - D_{\text{rad}}/D_{\text{rad}} \) where \( D_{\text{rad}} \) is the correlation value, \( D_{\text{eff}} \) is the corresponding data value, and \( N \) is the total number of data points in the correlation.
examples are of particular interest in the following discussions. (i) If the flow were quasi-steady, then $D$ would be independent of $\beta$. (ii) If the flow were highly turbulent, convective mixing would overwhelm the effects of molecular diffusion, $Sc$ would consequently have little effect, and $\kappa$ should be replaced by $\nu$. (iii) If the flow were both quasi-steady and highly turbulent we would expect $D$ to be independent of both $\beta$ and $Sc$.

Comparison to Other Experiments. Due to substantial differences between our experiments and those of Tarbell, et al. [19] and Scherer, et al. [13] (hereinafter referred to as Tarbell and Scherer, respectively), direct comparisons are difficult. The major difference is that while our measurements are local in the sense that we determine $D_{\text{eff}}$ from the concentration gradient across a single generation, both Tarbell and Scherer measured the rate of dispersion over the entire branching network and their results are therefore influenced to varying degrees by the local transport process in each individual generation.

Despite these differences, some useful comparisons can be made. In order to cast all the results in comparable form, we first note that the dispersion expressions obtained by Tarbell and Scherer represent a network average. We will assume that the dependence they observe on Reynolds or Peclet number (and $\alpha$ for Tarbell) is consistent throughout all generations. Accordingly, the rate of dispersion is greatest in the lower generations where the highest Reynolds numbers are encountered. The network dispersion coefficient that they define, therefore, can be thought of as a weighted average of the local values of $D_{\text{eff}}$. When casting their results in dimensionless form, Tarbell uses the total cross-sectional area of the second generation as the approximate "network area." In the following comparison, we assume that the network dispersion coefficients defined by Scherer and Tarbell are roughly equivalent to the dispersion coefficient in their second generations. While this is somewhat arbitrary, the error in the approximation is likely to be relatively small and will affect only the numerical constant (e.g., $a$ and $\bar{q}$ in equation (3)) and not the exponents. Based on these assumptions we will express the results of Scherer and Tarbell in terms of the Peclet number in the second generation (based on tube diameter).

We relate Tarbell's Reynolds number ($Re_T = U_c a/v$ where $U_c$ is the velocity amplitude in the parent tube of the network) to a local one using

$$Re_T = Re_{T}/2$$

Dimensionless dispersion coefficients are thus obtained for the results of Tarbell

$$D_T = 0.003553 (Pe)^{0.82} Sc^{-0.06}$$

and Scherer

$$D_S = 0.9 Pe$$

While we have introduced $Sc$ to equation (10) to be consistent with Tarbell's dimensionless formula, in the following comparisons $Sc$ is set equal to unity.

Within the range of Reynolds number and dimensionless frequency investigated in both unsteady experiments (which also overlap the range of Peclet number tested by Scherer) the comparison of Fig. 5 can be made for a gas mixture with $Sc = 1$. Note that in placing Tarbell's correlations on this graph, we have implicitly assumed that mixing by molecular diffusion is unimportant and therefore, that Tarbell's results are unaffected by Schmidt number. The validity of this assumption is considered later.

More recently, Azhar and Tarbell [2], using methods similar to those of Tarbell, have shown that their network dispersion coefficient $D_N$ takes on one of two forms:

$$D_N = Re^{1.43} a^{1.64}$$

for low $Re$, and

$$D_N = Re^{1.57}$$

for high $Re$. They found that the transition value of $Re$ depended on oscillation volume and ranged from 2000 to 9500 in their experiments.

The Effect of Unsteadiness. As noted earlier, the transport process is quasi-steady when $\tau_{rad}/T < 1$, leading to a result in which $D$ is found to be independent of $\beta$. Our expressions for $D$ show a clearly significant dependence upon $\beta$ (or $\alpha$), at least for values of $\beta > 4$ (see Table 1), suggesting that $\tau_{rad}/T > 1$, generally. The data for $\beta < 4$ are inconclusive in this regard due largely to a limited amount of data in this
range. By comparison, the results of Tarbell given in (11) yield a smaller, yet nonzero exponent for $\beta$.

However, the more recent data from Azhar and Tarbell show two distinct regimes depending on whether $Re$ is above or below some transition value. Their high $Re$ correlation is independent of $\alpha$ and therefore quasi-steady. The reason for this, presumably, is that turbulent mixing decreases the radial equilibrium time so that $\tau_{eq}/T$ becomes less than one. The low $Re$ correlation from Azhar and Tarbell bears a striking resemblance to ours (Table 1) in terms of the exponents on $Re$ and $\alpha$. In both cases the result appears to be intrinsically unsteady.

We can also compare our low frequency results to those of Scherer, et al. [13] for steady flow through a branching network. Taking all our data from both generations for $\beta < 4$, we find that the ratio $D_2/D_1$ has a mean value of 0.50 with a standard deviation of 0.30 for the 24 experimental points within this range of $\beta$. It has been suggested by Pedley [13] that a more appropriate unsteadiness parameter in a branching network is $\epsilon = (\Delta L/\Delta L)$ where $L$ is the generation length. Choosing those data for which $\epsilon < 1$ (37 data points) the mean value for $D_2/D_1$ is 0.48 $\pm$ 0.22. Thus, the approximate agreement between Scherer's findings and our low frequency results and the good numerical agreement indicated by the values for $D_2/D_1$ given in the foregoing, support our claim that these low frequency experiments are quasi-steady.

The Effect of Molecular Diffusion. Based on the comparison in Fig. 5, there appears to be reasonably close agreement between our results and those of Tarbell in view of the approximations on which the comparison is based. Even closer correspondence is found between our data and the low $Re$ data of Azhar and Tarbell. All this suggests that molecular diffusion plays at least a small role in the transport process of these experiments. This conclusion is consistent with the results of Knopp, et al. who found no significant effect of molecular diffusivity in the rate of tracer clearance from dog lungs during high-frequency oscillation.

There is reason to believe, however, that some differences might exist between liquids and gases, based on experimental results of dispersion under steady flow conditions, even when the flow is turbulent or in the presence of secondary flow. In a comparison between the theoretical prediction of Taylor [20] for axial dispersion under conditions of steady flow in a straight tube, and experimental results, Levenspiel [12] has attributed the observed differences to the effects of molecular diffusion. Gas and liquid experiments conducted in curved tubes with secondary flows also exhibit large differences in axial dispersion in the range $Re < 10^4$ (van Andel, et al. [3]). Thus, while the results obtained in branching tubes are not, by themselves sufficient to show a significant effect of Sc, experiments in other configurations suggest that some differences may exist between liquids and gases.

Concluding Remarks. Correlations have been presented that are applicable to models of high frequency ventilation in the lung. These demonstrate that tidal volume exerts a greater influence than frequency of oscillation, consistent with the measurements of gas exchange during HFV in test animals.

Our results show a significant dependence on dimensionless frequency demonstrating that, at least for Reynolds numbers up to 1000 and values of $\beta > 4$, the effects of unsteadiness are important. Our data lie remarkably close to the results for oscillatory flow in a straight tube but are universally higher. This enhancement of axial transport can be attributed to one or more of the following mechanisms: increased cross-stream mixing, a less uniform axial velocity profile (both of which enhance augmented dispersion), and convective streaming.

There remain, however, a number of uncertainties. Inclusion among these are questions pertaining to the importance of molecular diffusion both at low and high Reynolds number and the effect of and conditions for a transition from laminar to turbulent flow. In addition, the relative importance of streaming versus augmented dispersion in all the experiments conducted in branching networks needs to be further examined.

Acknowledgments

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References

Table 1  Data correlations

Coefficients are provided for two general expressions:

\[(D_{eff} - \kappa)/\kappa = q Pe^2 \beta^2\]

and

\[D_{eff} = \kappa q \gamma V_t \tau_r \beta\]

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(*) Computed from the expression \[(D_{eff} - D_0)/D_0)/N\] where \(D_0\) is the correlation value, \(D_{eff}\) is the corresponding data value, and \(N\) is the total number of data points in the correlation.