HEAT TRANSFER AUGMENTATION AND HYDRODYNAMIC STABILITY THEORY: UNDERSTANDING AND EXPLOITATION

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ABSTRACT

Resonant heat transfer enhancement in internal flows, based on excitation of flow instabilities, is the subject of this study. Exploitation of shear layer instabilities requires: creation of a system with separated flow; determination of the system natural frequency; and performance of appropriate excitation at that frequency.

Separated flows in transversely-grooved channels and in planar channels with transverse cylinders exhibit natural oscillations at Reynolds numbers above a critical value. At low Reynolds numbers, disturbances to these flows decay at frequencies close to those of the self-sustaining supercritical oscillations. It is shown that subcritical grooved channel modes may be "actively" excited by externally modulating the flow rate at their natural frequencies. They may also be "passively" sustained by placing cylinders in the system which shed vortices at those same frequencies. This flow destabilization is shown to generate significant transport enhancement. This technique is applicable to such devices as ribbed heat exchangers, electronic coolant systems, and biomedical blood oxygenators.

INTRODUCTION

Heat transfer processes frequently limit the size, performance and efficiency of practical engineering devices. Primarily for this reason, over the last several decades there has been an increasing interest in heat transfer enhancement. The first extensive survey on this subject appeared twenty years ago [1], and the most recent guide to the literature was assembled in 1985 [2].

We define augmentation as an increase of the convective heat transfer coefficient produced by some deliberate modification of a system. It is understood that this "enhancement" refers to improvement relative to the performance of a "standard," functionally similar system. This definition appears consistent with the past usage and convention [2], and allows for a meaningful
evaluation of the transport enhancement versus the increased pumping power requirement for the respective systems.

A broad range of augmentation schemes are used in everyday engineering practice. Proposed methods are usually grouped according to the type of hardware modification imposed on the standard system [2]. Another possible categorization is to divide various methods according to the physical effects that they produce. This latter classification leads to only a few generic categories. For single-phase systems we can identify the following two methods:

1. Modification of effective thermophysical properties - molecular and microscopic effects, such as additives schemes and electrostatic field application.

2. Increasing mixing - improving exchange of fluid normal to the heat transfer surface.

An important class of the second broad category is represented by mixing produced by hydrodynamic instability. In fact, many augmentation schemes are best understood and optimized in terms of concepts developed in hydrodynamic stability theory [6]. This class plays an important role in augmentation schemes both in terms of frequency of its current application and its potential for further development.

Turbulent flows represent common examples of mixing produced by hydrodynamic instability and we now discuss the mechanisms of their onset. In plane Poiseuille flow, finite amplitude disturbances are unstable to three-dimensional perturbations at Reynolds numbers, $Re$, greater than a transitional value of $Re_t \approx 1000$ [3]. While this flow is usually observed to become turbulent around $Re \approx 1000$, experiments in which external disturbances are held to a minimum demonstrate that laminar flow can be maintained up to $Re \approx 5772$ [4]. At that "critical" Reynolds number, $Re_c$, laminar plane Poiseuille flow exhibits two-dimensional oscillations, followed by a three-dimensional breakdown to turbulence [5].

In turbulent flow, fluid exchange at the wall is maintained by periodic burst disruptions of the viscous sublayer. These bursts are caused by instability of the layer itself. With an increase in Reynolds number, the frequency of these disruptions increases, resulting in stronger exchange of fluid near the wall. Although increasing the Reynolds number yields higher heat transfer coefficients, this process is not usually considered an augmentation method since it occurs "naturally," without any modification of the system. On the other hand, introduction of roughness on the heat transfer surface, which further destabilizes the sublayer and thereby locally intensifies the mixing process, could be considered a mixing augmentation scheme.

In systems with separated flows - a class of considerable interest in augmentation schemes and to practical applications - the flow behavior is markedly different than in planar-wall channel flows. With the presence of separation, the velocity profile develops an inflection point, making that region susceptible to Kelvin-Helmholtz shear layer instability. Under these conditions, the
critical Reynolds number marking the onset of natural, two-dimensional oscillations can be drastically reduced, falling well below the transition Reynolds number, \(Re_c < Re_t\) [6]. For these separated flow systems, at Reynolds numbers, \(Re\), such that \(Re_c < Re < Re_t\), two-dimensional self-sustained oscillation at a (well) defined frequency set in, driven by the Kelvin-Helmholz instabilities. The structure of these excited modes promotes lateral mixing in the flow, with a significant effect on system heat transfer performance. The onset of these oscillations does not require any external disturbances to the flow. For \(Re < Re_c\), flow disturbances decay in a damped oscillatory manner, with approximately the same frequency as the one dominating the self-sustained oscillations in the supercritical case [7,8].

We conclude that for separated flows, there exists "natural" modes which are oscillatory in character [7] and that their structures are useful for heat transfer enhancement. It has been shown that these modes can be excited by externally modulating the flow at their natural frequencies, not only for Reynolds numbers above the critical value [6,9], but also below it [6,8,9,10,11]. "Resonant enhancement" is the exploitation of this feature to augment large-scale lateral mixing and associated convective heat transfer.

Resonant enhancement can be very beneficial for transport systems with existing separated flows operating in the subcritical flow range. In such systems, the transport is often inhibited by very poor mixing in the external flow and very slow moving fluid in the separated region. Under certain hydrodynamic operating conditions, electronic equipment coolant systems, ribbed heat exchangers, and biomedical blood oxygenators are examples of such devices. Other useful applications might include mixing enhancement of viscous sublayers caused by deliberate modifications of wall the region, such as the machining of microgrooves in the walls, and subsequent modulatory excitation of the corresponding least stable modes. In some cases, modification of the wall may be useful in reducing the critical Reynolds numbers. In the next sections, examples demonstrating the main features of the resonant enhancement concept are presented.

CURRENT STUDY

Problem Definition

We consider two-dimensional, fully-developed flow in the periodically-grooved channel shown in Fig. 1. Cylinders of radius \(r_0\) are placed periodically in the channel such that their centers are located at \((x + jL, y)\), \(j\) integer. The presence of grooves and/or cylinders facilitates separated flow - the principal requirement for deployment of the resonant enhancement concept. The volume flow rate parallel to the \(x\)-direction, \(Q\), is specified as \(Q = Q[1 + \eta \sin (2\pi f_p \cdot t)]\). The hydrodynamics of this flow are parametrized by the following dimensionless numbers: the Reynolds number, \(Re = \bar{U} h/\nu\), the oscillatory fraction of the flow rate, \(\eta\), and the forcing Strouhal number, \(\bar{f}_p h / \bar{U}_0\).
FIGURE 1. A schematic of the periodically grooved channel geometry. Periodically placed cylinders of radius \( r_c \) are located with centers at \((x_c + jL, y_c)\), \( j \) integer. The steady flow rate component is in the positive \( x \)-direction.

The thermal conditions are uniform heat flux from the grooved wall, an adiabatic flat wall, and a fully developed and periodic temperature profile \([8]\). The working fluid Prandtl number is \( \text{Pr} = \gamma \). We define the groove average Nusselt number, \( \text{Nu} = qh/\langle \Delta T_b \rangle k \), where \( q \) is the total heat input from one groove periodicity length divided by its projected area, \( \Delta T_b \) is the difference between the wall temperature and the bulk fluid, and \( \langle \Delta T_b \rangle \) is its average over one channel periodicity length. In the remainder of this work, all lengths are normalized by \( h \), all times by \( h/U_c \), and all temperature differences by \( qh/k \).

Our objectives are to: 1) determine the geometric and hydrodynamic conditions for the onset, and the resulting frequency, of natural oscillations in steadily forced flow, \( \eta \approx 0 \), 2) show that these oscillations can be deliberately excited in subcritical flows by forced flow rate unsteadiness, or by other means, and 3) quantify the resulting resonant enhancement of heat transfer.

Methodology

Both experimental and numerical results are cited in this investigation. A diagram of the grooved channel test section used in the experimental part of this study is shown in Fig. 2. The flow rate is produced by a centrifugal mean flow pump and a Scotch-yoke oscillatory pump. Water, at 20°C (\( \text{Pr} = 9 \)), enters the grooved test section. For the values of \( \text{Re} \) investigated, the flow is fully developed by the time it reaches the seventh and eighth grooves, where flow visualizations and heat transfer measurements are made. The ratio of the channel width to the channel half height is 25, and visualizations indicate that in laminar flow, center channel flow structure is essentially two-dimensional.

Electrical film heaters are bonded to the grooved wall, in the region indicated in Fig. 2, including the vertical portions. To minimize buoyancy effects, the heated surface is placed on the gravitational top of the test section, and the ratio \( \text{Gr}/\text{Re}^2 \), where \( \text{Gr} \) is the Grashoff number, is kept small. The average of the
temperature differences between the sixteen wall locations shown in Fig. 2 and the fluid bulk temperature are used in calculating the Nusselt number. The dimensionless geometric parameters used for much of the experimental study are $L = 10.41$, $l = 3.42$, $a = 1.71$, $h = 1$, and $r_e = 0$, and this geometry is referred to as GE. Further details of the experimental set-up are given in [6].

The two-dimensional Navier–Stokes and energy equations are solved numerically in one periodicity length of the domain shown in Fig. 1 using the spectral element method [12–15]. The full nonlinear heat transfer calculations have been performed for the case $L = 6.67$, $l = 2.22$, $a = 1.11$, $h = 1$ and $r_e = 0$, which is referred to as geometry GN. Simulations of forced convection heat transfer are done for Prandtl numbers in the range $1 \leq Pr \leq 5$ [8], and the results are extrapolated to $Pr = 7$, [11].

RESULTS

Grooved Channels Without Cylinders

Steadily forced flow. An experimental ink visualization in geometry GE at $Re = 525$, shown in Fig. 3, is observed to be steady. To visualize the groove vortices, ink is injected directly into the groove. We can infer that this flow is subcritical and that its convective heat transfer is limited by poor channel mixing and stagnant groove vortices. Visualizations in which no ink is injected directly into the groove are shown in Fig. 4 for $Re = 525$, 750 and 1500. Flow field oscillation are first observed at a critical Reynolds number of $Re_c = 750$ [6]. This is most easily
FIGURE 3. Experimental ink flow visualization, Re = 525, η = 0, geometry GE. Flow is from right to left. Downward motion of the ink is due to a specific gravity slightly greater than unity. Some ink is injected directly into the groove to visualize the primary vortex. The transport character of this subcritical flow is limited by stagnant groove vortices and a lack of external channel mixing.

seen in Fig. 4 by the presence of ink in the groove. As Re is increased further, the flow field shows its first sign of small scale, three-dimensional mixing at a transition Reynolds number of Reₜ = 0(1000), and it is clearly turbulent at Re = 1500. Periodic large scale fluid motion persists into the turbulent flow regime.

Fig. 5 shows the streamlines of a numerical simulation of the flow in geometry GN at Re = 525. The flow field is seen to be steady at this Reynolds number and have very similar characteristics to the subcritical experimental visualization shown in Fig. 3. Further simulations show that the velocity field in GN is steady for Reynolds numbers up to Reₐ = 975 [7].

The numerically calculated streamwise velocity as a function of time, at a "typical" point in the supercritical flow at Re = 1200 in geometry GN, is shown in Fig. 6a [7]. We see that at this large Reynolds number, the two-dimensional velocity field is not steady, even under steadily forced conditions, but experiences natural sinusoidal oscillations at a frequency of \( \Omega_n = 0.142 \).

Instantaneous streamlines of this flow field are shown at eight different times of the oscillatory period in Fig. 7. Supercritical mode excitation causes strong groove vortex motion and a small but noticeable amount of waviness to the external channel flow.

The temporal behavior of the streamwise velocity in GN, for a subcritical flow at Re = 800, following a small flow disturbance is shown in Fig. 6b [7]. The disturbance is shown to decay with time with a natural frequency very nearly the same as that of the self-sustaining oscillations of the supercritical flow. The oscillatory
FIGURE 4. Experimental ink flow visualizations, $Re = 525$, 750 and 1500 and $\eta = 0$, geometry GE. The onset of natural oscillations is observed at $Re_c = 750$, while a break down to turbulence appears at $Re_a = 0(1000)$. The dark circular arc in the upper left hand groove corner is a wire located on the back channel wall, far from the $x,y$-plane being visualized.

The behavior of these two flows has been shown to be the result of natural stability modes of the grooved channel flow [7]. At subcritical Reynolds numbers, these modes are damped. In supercritical flow, however, energy is projected from the steady forcing onto these modes and sustains them.
FIGURE 5. Numerical streamlines, Re = 525, η = 0, geometry GN. The stable flow field is found to be steady.

(a)

FIGURE 6. Temporal plot of streamwise velocity at a "typical" point of flows in geometry GN.

a Re = 1200, η = 0. Self-sustained oscillation, supercritical flow, Ω = 0.142.

b Re = 800, η = 0. Damped sinusoidal response of a small disturbance to a subcritical flow.
The natural oscillations of supercritical flows are triggered by Kelvin-Helmholtz instabilities of the groove-spanning free shear layers. Free shear layers are unstable to perturbations which are sufficiently long compared to the layer's thickness. In grooved channels, the perturbation length scales which can interact with...
the groove shear layers are geometrically constrained to be less than or equal to the groove length, \( l \). At low Reynolds numbers, the shear layer is relatively thick, and the short admissible shear layer perturbations cannot destabilize it. As the Reynolds number increases, however, the thickness of the groove free shear layer decreases and it becomes unstable to shorter perturbation modes. The growth rate of shear layer disturbances increases with the magnitude of the velocity difference across the layer, which also increases with Reynolds number. At sufficiently large Reynolds numbers, the admissible shear layer instabilities are energetic enough to drive the Orr-Sommerfeld modes of the external channel flow, triggering flow oscillation. The natural frequency of the resulting flow field has been shown to be governed by the same dispersion relation as planar channel Orr-Sommerfeld modes \([7]\). A method for predicting the natural frequency of these flows for a range of grooved channel geometries is presented in \([7,11]\).

The critical Reynolds number for the onset of laminar oscillations has been shown to be a function of the channel geometry. The dependence of the critical Reynolds number on the geometric ratio, \( l/L \), is shown in Fig. 8. This curve includes data for flat channels, periodically-grooved channels with rectangular and semi-circular grooves, from both experimental \([6,16]\) and numerical

![Graph showing the dependence of Re_c and Re_t on channel geometry. Both experimental data (circles) \([6,16]\) and two-dimensional numerical results (triangles) \([7,17]\) are shown. For "long" grooves, \( l/L > 0.3 \), natural oscillations set in at lower Reynolds numbers than that for the transition to turbulence, allowing natural, laminar oscillations to be observed experimentally, as in Fig. 4.](image)

**FIGURE 8.** Dependence of Re_c and Re_t on channel geometry. Both experimental data (circles) \([6,16]\) and two-dimensional numerical results (triangles) \([7,17]\) are shown. For "long" grooves, \( l/L > 0.3 \), natural oscillations set in at lower Reynolds numbers than that for the transition to turbulence, allowing natural, laminar oscillations to be observed experimentally, as in Fig. 4.
[7,17] investigations. We see that as the ratio $l/L$ increases, $Re$ drops rapidly. As $l/L$ increases, smaller Reynolds number flows are able to produce admissible perturbations which are energetic enough to destabilize the groove shear layers. The groove shape (rectangular or semi-circular), however, has little effect on $Re$. Further investigations are required to determine the dependence of $Re_c$ on the parameter $l$.

The experimentally determined transition Reynolds number, $Re_t$, is also included in Fig. 8 [6], and is seen to be relatively insensitive to channel geometry [18]. This transition is triggered by a "universal" secondary instability mechanism similar to that observed in flat channels. For "long" grooves, $l/l > -0.3$, natural oscillations set in at lower Reynolds numbers than that for the transition to turbulence. Under these conditions, natural, laminar oscillations may be observed experimentally, as in Fig. 4. For short grooved channels, $l/l < -0.3$, as in flat channels, these laminar oscillations cannot be seen experimentally.

**Unsteadily forced flow.** We wish to investigate resonant enhancement by superimposing small amplitude flow rate modulation on naturally steady, subcritical flows. We consider unsteadily forced flows at $Re = 525$, $\eta = 0.2$ and a range of $\Omega_p$. In both the experimental and numerical geometries, GE and GN, flows at $Re = 525$ are subcritical.

Fig. 9 shows flow visualization for which $Re = 525$, $\eta = 0.2$ in geometry GE at three different forcing frequencies, $\Omega_p = 0.026$, 0.077 and 0.307. While the high and low frequency cases show some communication between the grooves and the channel, they do not show the marked increase in channel mixing demonstrated in the intermediate frequency case. The natural frequency of this flow’s most unstable mode has been shown to be $\Omega_p = 0.080$ [6,11]. The peaked enhanced mixing near this natural frequency demonstrates resonant excitation.

Instantaneous streamlines are shown in Fig. 10 for $Re = 525$, $\eta = 0.2$ and $\Omega_p - \Omega_c = 0.142$ in GE at six different times during the oscillatory cycle. We see that the forced oscillations produce strong groove vortex motion and impart a traveling wave structure on the external channel flow. As this wavey structure passes over a given location of the wall, fluid is alternately brought to and taken away from the surface. We therefore expect that excitation of the system’s most unstable mode leads to significant transport enhancement.

To quantify the flow field transport response to forced oscillation, we define the heat transfer enhancement factor, $E$, as the ratio of the groove averaged Nusselt number under unsteadily forced conditions, $\eta > 0$, to that for steadily forced flow, $\eta = 0$. Plots of $E$ versus $\Omega_p$ for $Re = 525$ and $\eta = 0.2$, from both experimental measurements and numerical calculation, are shown in Figs. 11 and 12, respectively. We see that the enhancement ratio exhibits a strong resonant character. The factors of two and a half heat transfer enhancement gained by a small (20%) flow rate oscillation is striking. It is seen that by exploiting the natural modes of the grooved channel system, a properly tuned small amplitude flow rate modulation produces a very large flow field and transport response.
\( N_F = 0.026 \)

\( N_F = 0.077 \)

\( N_F = 0.307 \)

FIGURE 9. Experimental unsteadily forced flow visualizations for \( Re = 525, \eta = 0.2, N_F = 0.026, 0.077 \) and 0.307. The peaked mixing response near the system's natural frequency of \( N_n = 0.080 \) demonstrates resonant excitation.

The important question of pressure drop is considered numerically in [19]. The initial results indicate that at the onset of large-scale mixing, produced either by external flow rate modulation, \( \eta > 0 \), or spontaneously in supercritical flow, \( Re > Re_c \), the pressure drop increases dramatically. The relative increase in pressure drop, however, is less than the corresponding heat transfer enhancement.

Planar Channel With Cylinders

In this section, numerical results are presented for steadily forced planar channel flow in which flow separation is maintained
FIGURE 10. Numerically calculated instantaneous streamlines at six different time during the forced oscillatory cycle, for $Re = 525$, $\eta = 0.2$ and $\Omega_F = \Omega_m = 0.142$, in geometry GN. Resonant mode excitation causes strong groove vortex motion and impart a wavey structure to the external channel flow.

FIGURE 11. A plot of the experimentally measured groove averaged heat transfer enhancement factor, $E$, for $Pr = 7$, $Re = 525$, $\eta = 0.2$ and a range of $\Omega_F$, in geometry GE. A resonant response is demonstrated.
FIGURE 12. A plot of the numerically calculated groove averaged heat transfer enhancement factor, $E$, for $Pr = 7$, $Re = 525$, $\eta = 0.2$ and a range of $\eta_r$, in geometry GE. A resonant response is demonstrated.

by periodically placed cylinders. The basic geometry studied in this section, referred to as GC, is a planar channel, $l = a = 0$, containing cylinders of radius $r_c = 0.2$, which are spaced at the same periodicity length as that of geometry GN, $L = 6.67$. We consider a range of spacing between the cylinders and the channel centerline, $y$. For $y_c = 0.5$, Figs. 13a and 13b show the instantaneous streamlines of two flows at Reynolds numbers of $Re = 100$ and 225, respectively. The flow at $Re = 100$ is shown to be

(a)

FIGURE 13. Numerically calculated instantaneous streamlines in geometry GC.

a $Re = 100$, $\eta = 0$. Steady, subcritical flow.
b $Re = 200$, $\eta = 0$. Self-sustained oscillation, supercritical flow, $\eta_r = 0.185$. 
steady while that at Re = 225 is oscillatory, with a natural frequency of \( \Omega = 0.185 \). We see that this separated flow demonstrates the same type of subcritical-supercritical behavior as the grooved channel. The two waves per periodicity length structure demonstrated in Fig. 13b is very similar to that of the forced oscillatory flow in Fig. 10.

The addition of these periodic cylinders to a planar channel reduces the critical Reynolds number from 5772 to 125. The shear layers of the cylinder wakes in this geometry is about equal to the channel periodicity length, L. In grooved channels, Fig. 8 shows that as the groove shear layer length approaches the channel periodicity length, \( L/L \sim 1 \), the critical Reynolds number also approaches the value of 125. The critical Reynolds number in geometry GC is effectively insensitive to \( y_o \); it does, of course, increase as the cylinder radius, \( r_o \), decreases.

The natural frequency of flows in geometry GC at Re = 225 is dependent on \( y_o \). This "tuning" behavior is shown in Fig. 14, and it is seen that as the periodic cylinders are moved toward the center of the channel, the natural frequency of the flow oscillations increases. The existence of these natural oscillations at very low Reynolds numbers and their "tunability" suggest that natural grooved channel modes may be internally triggered using properly placed cylinder, rather than externally forced by flow rate modulation. This possibility would eliminate the need of an unsteady prime mover in practical applications, and is investigated in the next section.

**FIGURE 14.** Flow field natural frequency tuning relation, \( \Omega_n \) versus \( y_o \) in geometry GC at Re = 225.
Grooved Channels With Cylinders

It has been shown that natural modes of subcritical grooved channel flows are susceptible to resonant excitation by forced flow rate modulation, and the resulting "external" destabilization greatly enhances the transport character of the flow. In this section, we demonstrate that these subcritical oscillations may be triggered "internally" by adding properly placed cylinders to the channel which produce flow oscillations at the natural frequency of the grooved channel modes.

The natural frequency of the subcritical grooved channel flow at Re = 525 in geometry GN is \( \Omega_n = 0.142 \). At Re = 225, a cylinder placed in a planar channel at \( y_n = 0.5 \) produces natural flow oscillation at a frequency close to this value, \( \Omega_n = 0.185 \) (Fig. 14). Under these conditions in geometry GC, the resulting naturally occurring channel flow structure (Fig. 13b) is very similar to that of the forced oscillatory flow shown in Fig. 10. Fig. 15 shows the instantaneous streamlines of a steadily forced flow at Re = 525 in

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**FIGURE 15.** Numerically calculated instantaneous streamlines at five different times during the oscillatory cycle, for Re = 525 and \( \eta = 0 \) in geometry GN with a cylinder of radius \( r = 0.2 \) at \( (x_n, y_n) = (0, 0.5) \). Properly placed cylinders are shown to excite grooved channel modes.
a grooved channel whose dimensions are the same as that for geometry GN, except for the addition of periodic cylinders of radius \( r = 0.2 \) located at \( y = 0.5 \) and \( x = 0 \). Even in the absence of external forced flowrate modulation, the flow field shown in Fig. 15 is seen to exhibit oscillations whose structure is very similar to that of the unsteadily forced flow shown in Fig. 10. It is believed that further heat transfer investigations will show that this passive excitation technique will produce enhancement factors similar to those of forced oscillatory flow shown in Figs. 11 and 12. Furthermore, preliminary results indicate that these oscillatory "tuned" laminar flows can achieve transport rates comparable to those of high Reynolds number turbulent flow at a fraction of the dissipation cost.

**CONCLUSIONS**

The separated flows in grooved channels, and planar channels with cylinders, are shown to exhibit natural oscillation at sufficiently large, supercritical Reynolds numbers. In the subcritical regime, disturbances to the flow decay as damped sinusoids whose frequency is close to that of the self-sustaining supercritical oscillations. The value of the critical Reynolds number in these separated channel flows is shown to be a strong function of channel geometry.

Subcritical grooved channel modes may be "actively" excited by externally modulating the flow rate at the modal natural frequency. They may also be "passively" or "internally" sustained by adding periodic cylinders which produce flow oscillations whose natural frequency is close to that of the grooved channel flow. The passive method effectively reduces the critical Reynolds number for the onset of naturally self-sustained oscillatory flow.

Flow destabilization is shown to be a promising transport enhancement technique which can be applied to such devices as ribbed heat exchanger, biomedical blood oxygenators, and electronics coolant system. They not only provide high transport rates at low flow rates, but also appear to be more economical with regards to pumping power (dissipation).

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Groove depth (Fig. 1).</td>
</tr>
<tr>
<td>( E )</td>
<td>Dimensionless heat transfer enhancement factor.</td>
</tr>
<tr>
<td>( f )</td>
<td>Oscillatory frequency.</td>
</tr>
<tr>
<td>( h )</td>
<td>Channel half-height (Fig. 1).</td>
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<tr>
<td>( k )</td>
<td>Thermal conductivity.</td>
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<tr>
<td>( l )</td>
<td>Groove length (Fig. 1).</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>L</td>
<td>Channel periodicity length (Fig. 1).</td>
</tr>
<tr>
<td>Nu</td>
<td>Groove-averaged Nusselt number: ( qh/\Delta T_b k ).</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number: ( \nu/\alpha_T ).</td>
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<tr>
<td>q</td>
<td>Wall heat flux.</td>
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<tr>
<td>Q</td>
<td>Volume flow rate through channel.</td>
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<tr>
<td>Q'</td>
<td>Volume flow rate through channel per unit length normal to the plane of Fig. 1.</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number: ( \bar{U}_c h/\nu ).</td>
</tr>
<tr>
<td>Re_c</td>
<td>Re at the onset of laminar oscillations.</td>
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<tr>
<td>Re_t</td>
<td>Re at the onset of turbulence.</td>
</tr>
<tr>
<td>t</td>
<td>Time.</td>
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<tr>
<td>( \bar{U}_c )</td>
<td>Equivalent flat channel centerline velocity, parabolic profile: ((3/2)(\bar{Q}'/2h)).</td>
</tr>
<tr>
<td>u, v</td>
<td>Velocity components (Fig. 1).</td>
</tr>
<tr>
<td>x, y</td>
<td>Coordinate system directions (Fig. 1).</td>
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<tr>
<td>( \alpha_T )</td>
<td>Thermal diffusivity.</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Oscillatory fraction of the flow rate.</td>
</tr>
<tr>
<td>( \Delta T_b )</td>
<td>Local temperature difference between the wall and the bulk fluid.</td>
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<tr>
<td>( \langle \Delta T_b \rangle )</td>
<td>( \Delta T_b ) averaged over one channel periodicity length.</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Dimensionless frequency: ( fh/\bar{U}_c ).</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity.</td>
</tr>
</tbody>
</table>

**Subscripts**

- ( )_n Property of the most unstable mode.
- ( )_F Forced flowrate conditions.

**REFERENCES**


17. Amon, C., private communication.


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