SIMULATIONS OF THREE-DIMENSIONAL FLOW AND AUGMENTED HEAT TRANSFER IN A SYMMETRICALLY GROOVED CHANNEL WITH CONSTANT TEMPERATURE WALLS

M. Greiner, R.J. Faulkner, and R.A. Wirtz
Mechanical Engineering Department
University of Nevada
Reno, Nevada 89557

P.F. Fischer
Division of Applied Mathematics
Brown University
Providence, Rhode Island 02912

ABSTRACT

Direct numerical simulations of three-dimensional flow and augmented convection in a channel with symmetric, transverse grooves on two opposite walls are performed using the spectral element technique. The flow is driven by a constant pressure gradient. A method employing an exponentially decaying temperature scale is developed and used to calculate the fully developed heat transfer coefficient for constant temperature boundary conditions. Results are presented for the Reynolds number range 180 < Re < 1175.

A series of flow transitions is observed as the Reynolds number is increased, from steady two-dimensional flow, to traveling two and three-dimensional wave structures, and finally to three-dimensional mixing. Periodic ejection of slow moving fluid from the grooves causes significant flow rate unsteadiness. Three-dimensional simulations predict friction factor and Nusselt number values to within 20% of measured values over the narrow Reynolds number range where overlapping data exists. Two-dimensional simulations are found to be inadequate to calculate transport in this channel for Re > 400.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Groove depth, Figure 1.</td>
</tr>
<tr>
<td>b</td>
<td>Channel periodicity length, Figure 1.</td>
</tr>
<tr>
<td>c</td>
<td>Decay constant.</td>
</tr>
<tr>
<td>D_h</td>
<td>Minimum hydraulic diameter, 2H.</td>
</tr>
<tr>
<td>f</td>
<td>Fanning friction factor, ((f_0D_h)/((2U_0)^2)).)</td>
</tr>
<tr>
<td>f_x</td>
<td>Fluid body force per unit mass in the x-direction.</td>
</tr>
<tr>
<td>H</td>
<td>Minimum channel wall to wall spacing, Figure 1.</td>
</tr>
<tr>
<td>k</td>
<td>Fluid thermal conductivity, 0.0263 W/m°C.</td>
</tr>
<tr>
<td>K</td>
<td>Number of spectral elements.</td>
</tr>
<tr>
<td>n'</td>
<td>Wall unit normal.</td>
</tr>
<tr>
<td>N</td>
<td>Spectral element order.</td>
</tr>
<tr>
<td>Nub</td>
<td>Bulk Nusselt number based on projected area.</td>
</tr>
<tr>
<td>Nuc</td>
<td>Center point Nusselt number.</td>
</tr>
<tr>
<td>Pr</td>
<td>Fluid molecular Prandtl number, 0.70.</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number, UD_h/v.</td>
</tr>
<tr>
<td>Re_a</td>
<td>Time average Reynolds number, (U_aD_h/v).</td>
</tr>
<tr>
<td>t</td>
<td>Time.</td>
</tr>
<tr>
<td>T</td>
<td>Temperature.</td>
</tr>
<tr>
<td>T_b</td>
<td>Bulk temperature.</td>
</tr>
<tr>
<td>T_c</td>
<td>Center point temperature.</td>
</tr>
<tr>
<td>u, v, w</td>
<td>Velocity components in the x, y and z directions.</td>
</tr>
<tr>
<td>U</td>
<td>Mean x-velocity at the minimum channel cross-section.</td>
</tr>
<tr>
<td>U_a</td>
<td>Time average U.</td>
</tr>
<tr>
<td>V</td>
<td>Volume.</td>
</tr>
<tr>
<td>V_f</td>
<td>Volume flow rate.</td>
</tr>
<tr>
<td>W</td>
<td>Width of the computational domain, Figure 1.</td>
</tr>
</tbody>
</table>

Greek

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Thermal diffusivity, 2.63 \times 10^{-5} m^2/s</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Fluid kinematic viscosity, 1.84 \times 10^{-5} m^2/s</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Periodic Temperature.</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Fluid Density, 1.006 kg/kg.</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Period of local time variations.</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>Computation domain.</td>
</tr>
</tbody>
</table>

INTRODUCTION

Experimental flow visualizations have shown that cutting a continuous set of transverse grooves into one surface of a rectangular cross section passage excites secondary flows that enhance mixing (Greiner et al. 1989). In these experiments, the groove length and depth are of the same order as the minimum wall to wall spacing. Two-dimensional waves appear at Reynolds numbers as low as Re = 350, followed by an early transition to three-dimensional mixing.

Heat transfer and friction factor measurements using air as the working fluid have been performed to quantify the enhanced transport in single-grooved-wall passages (Greiner et al. 1991). These heat transfer measurements employed a uniform heat flux from the flat
surface, and a constant temperature on the grooved boundary. The measured heat transfer coefficient is based on the average temperature difference between the two surfaces, and hence quantifies heat transfer augmentation from one surface to the other due to enhanced mixing. After the onset of unsteady flow, both the Nusselt number and friction factor are greater than the corresponding values for fully developed flat channel flow. However, the heat transfer level for a given pumping power is greatly enhanced.

Direct numerical simulations of three-dimensional flow and heat transfer in the same single-grooved-wall passage were performed using the spectral element method by Greiner et al. (1996). The Fanning friction factor and Nusselt number versus Reynolds number relationships predicted by these simulations were in good agreement with the earlier experiments. Moreover, two-dimensional simulations were found to be inadequate to accurately capture the transport processes in this channel for Reynolds numbers greater than Re = 570.

In the current work, we consider symmetrically grooved channels (with grooves on both surfaces) with constant temperature boundary conditions. The double-grooved geometry is examined to determine whether flow destabilization is intensified compared to single-grooved-wall channels. Moreover, constant temperature wall conditions quantify the heat transfer from the passage to the fluid, and hence are more realistic than the boundary conditions employed in the earlier work for assessing the applicability of grooved channels to heat exchanger applications.

This work is being performed in parallel with an experimental study (Wirtz et al. 1997). That work uses a holographic interferometer to measure local heat transfer in the fully developed region of a passage with seven symmetric grooves. These studies have two primary goals. The first is to determine if grooved channels are competitive with other enhanced heat exchanger core geometries which use air as the working fluid (Prandtl number Pr = 0.7). The second is to develop spectral elements techniques and meshes to calculate the transport properties of these passages, and then evaluate the accuracy of these calculations with direct comparisons with experimental data.

The next section describes the computational methods and boundary conditions employed in this work. A technique for obtaining periodically fully developed heat transfer results in a constant wall temperature condition is described in detail. The evolution of the velocity and temperature fields as Reynolds number is increased is then presented. Finally, heat transfer and friction factor results from these simulations are presented and directly compared to experimental results.

**NUMERICAL METHOD**

**Computation Domain**

Figure 1 shows the primary three-dimensional computational domain and spectral element mesh used in this work. The upper and lower V-shaped boundaries are solid walls. The flow is driven from left to right through the computation domain by a uniform body force per unit mass, $\mathbf{f}_g$. This body force is equivalent to a constant axial pressure gradient, i.e. $f_x = -(\partial p/\partial x)/\rho$. This domain is the same as that of the earlier studies of a channel with one grooved wall (Greiner et al. 1991, 1996), except grooves are now placed on both the top and bottom surfaces. The dimensions $a = 1.2$ cm, $b = 2.4$ cm, and $H = 1.0$ cm, are identical to the lengths employed in the companion experimental study (Wirtz et al. 1997).

Periodicity is imposed on the boundaries at $x = 0$ and $b$ so these simulations model flows which are periodic from groove to groove. Periodicity is also imposed on the boundaries at $z = 0$ and $z = -W$.

![Figure 1 Computational domain and spectral element mesh.](image)

where $W = 2.0$ cm. The computational grid shown in Fig. 1 consists of four levels in the z-direction, each containing 160 hexahedral spectral elements (Patera 1984, Maday and Patera 1989), for a total of $K = 640$ elements. The mesh is more refined near the peaks of the grooves and along the interface between the grooves and the open channel region than in the rest of the domain. Within each element, the solution, data and geometry are represented by tensor-product bases polynomials of degree $N$ in each spatial direction, corresponding to a total grid point count of roughly $KN^3$. Numerical convergence is achieved by increasing the spectral order, $N$, at the fluid body force $f_X$ and the resulting Reynolds number are increased. Two variations of the mesh are used to demonstrate grid independence. A mesh with six z-levels instead of four is used to determine if a more refined grid affects the results. Another mesh with a larger width $W = 3.0$ cm, and six z-levels is used to determine if the results are affected by a wider domain with the same grid point spacing as the baseline case.

Two-dimensional simulations are performed using a planar grid that is equivalent to the face of the domain shown in Fig. 1. These two-dimensional calculations are used as initial conditions for low-Reynolds-number three-dimensional cases by replicating the flow and temperature fields at each z-station, and setting the initial z-velocity, $w$, to zero.

The present simulations use a fractional step (or time splitting) approach (Orszag and Kells 1980; Fischer and Patera 1992). In this technique, the coupled momentum equations are advanced by first computing the convection term, followed by the projection of the velocity onto a divergence-free space, and then a viscous update which incorporates the velocity boundary conditions. At each time step the x-velocity component, $u$, is integrated throughout the domain $\Omega$, to determine the instantaneous volume flow rate per unit width.
\[ V_r W = (1/LW) \int_0^L \int_0^1 \nabla \cdot \nabla \theta \, \mathrm{d}x \, \mathrm{d}y. \]  

The integral \( V_r W = (1/L) \int_0^L \nabla \cdot \nabla \theta \, \mathrm{d}A \), is used for two-dimensional simulations, where \( A \) is the area of the planar domain.

In general, the velocity field and the integrated volume flow rate may vary with time, even under a steady body force. The resulting time dependent Reynolds number is defined as \( Re = \frac{UD_b V}{\nu} \), where the time-dependent velocity through the minimum channel cross-section is \( U = \frac{V_r W}{W} \), and the minimum hydraulic diameter is \( D_h = 2H \). Simulations are run until a "steady state" solution is attained. At low Reynolds numbers, steady state is time-independent. As the Reynolds number is increased, steady state conditions first exhibit time periodic volume flow rates, and then at higher Reynolds numbers, steady state is defined when the flow rate randomly varies about a "constant" mean value. When the flow is at steady state, the time averaged Reynolds number, \( Re_{\text{av}} = U_{\text{av}} D_h / \nu \), and time averaged Fanning friction factor \( f = \left( \frac{x D_h}{2U_{\text{av}}^2} \right) \), are calculated, based on the time average velocity \( U_{\text{av}} \). At higher Reynolds numbers, where the time variation of \( U \) is irregular, it is difficult to determining how long simulations need to run to calculate an accurate value of \( U_{\text{av}} \).

**Constant Temperature Boundary Conditions**

The solution technique for computing the fully developed temperature field for constant temperature boundary conditions follows the analysis of Patankar et al. (1977). The energy equation, as well as the initial and boundary conditions solved in the computation domain \( \Omega \), are

\[ \frac{\partial \theta}{\partial t} + \nabla \cdot \mathbf{U} \theta = \frac{\partial}{\partial x} \left( \alpha \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left( \alpha \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( \alpha \frac{\partial \theta}{\partial z} \right) \]  

(1a)

\[ T(x,y,z,t=0) = T_{\text{init}}(x,y,z) \]  

(1b)

\[ T(x,y,z,t) = 0 \text{ on the walls} \]  

(1c)

\[ T(x=b,y,z,t) = C T(x=0,y,z,t) \]  

(1d)

Equation 1d corresponds to the "fully developed" condition where the temperature profile is self-similar from one cell to the next, i.e., \( T(x+b,y,z,t) = C T(x,y,z,t) \) for all \( (x,y,z) \). The constant \( C \) is unknown and is a parameter to be determined as part of the computation. The fact that each cell independently satisfies the homogeneous equation (1) and that we are considering fully developed solutions which are independent of \( T_{\text{init}} \) implies that the solution to (1) for each cell would yield the same value of \( C \). Hence, \( C \) cannot be a function of \( x \). Moreover, it is readily demonstrated from energy arguments that under fully developed conditions \( C \) cannot be a function of \( x \) even if the flow is initially.

Any function satisfying the above self-similar condition has the unique decomposition \( T(x,y,z,t) = e^{-ct} \Theta(x,y,z,t) \) where \( \Theta(x+b,y,z,t) = \Theta(x,y,z,t) \) is a periodic function and \( c \) is defined to satisfy \( C = e^{ct} \). Thus, the computation of \( \Theta \) is reduced to the computation of a periodic function \( \Theta \), and a constant \( c \). Substituting this decomposition into equation 1 yields:

\[ \frac{\partial \Theta}{\partial t} + U \cdot \nabla \Theta - \alpha \cdot \nabla^2 \Theta = (\alpha \cdot c^2 + uc) \Theta - 2\alpha \cdot c \frac{\partial \Theta}{\partial x} \]  

(2a)

\[ \Theta(x,y,z,t=0) = \Theta_{\text{init}}(x,y,z) \]  

(2b)

\[ \Theta(x,y,z,t) = 0 \text{ on the walls} \]  

(2c)

\[ \Theta(x=b,y,z,t) = \Theta(x=0,y,z,t) \]  

(2d)

Since the fully developed solution is independent of the initial condition we may arbitrarily set \( \Theta_{\text{init}} = 1 \). Equation 2a is solved using a semi-implicit time-stepping procedure similar to that for our Navier-Stokes solver. The diffusive terms are treated implicitly while the convective terms are treated explicitly. In addition, all terms on the right of Equation 2a are treated explicitly using the latest available value for \( c \).

In the steady state case \( (\partial \Theta / \partial t = 0) \), Equation 2 constitutes an eigenproblem for the eigenpair \( (c, \Theta) \). The constant \( c \) corresponds to the decay rate of the mean temperature in the \( x \)-direction. A larger value of \( c \) implies more rapid decay and more effective heat transfer. In the convection-dominated limit where the Peclet number \( U_{\text{av}} \alpha / \nu \) is large, Equation 2a becomes a linear eigenvalue problem. In this case standard iterative methods for computing the lowest value of \( c \) (corresponding to the most slowly decaying mode in \( x \)) can be used even when the nonlinear \( \Theta^2 \) term in Equation 2a is not identically zero. We find that this method accurately computes the decay rate and Nusselt numbers for steady flows in square and round ducts (Kays and Crawford 1991).

For steady periodic flows with period \( \tau \), the temperature is periodic in time, implying \( T(x,y,z,t+\tau) = T(x,y,z,t) \). Since \( c \) is independent of time, this implies that \( \Theta(x,y,z,t+\tau) = \Theta(x,y,z,t) \). If the value of \( c \) is not chosen correctly, this condition will not be satisfied. Unfortunately, \( \tau \) is not known a priori but is a result of the hydrodynamic part of the calculation. A robust approach to compute \( \Theta \) and \( c \) is obtained by multiplying Equation 2a by \( \Theta \), integrating over a single cell \( \Omega \), and simplifying to yield:

\[ \frac{1}{2} \frac{d}{dt} \int_\Omega \Theta^2 \, dV = \int_\Omega \left[ ((\alpha \cdot c^2 + uc) \Theta^2 - \alpha \cdot \nabla \Theta \cdot \nabla \Theta) \right] \, dV \]  

(3)

While we do not expect the time derivative of the average temperature (represented by the left-hand side of Equation 3) to be identically zero, it will in general be less than the time derivative of \( \Theta \) at any one point in the domain. Moreover, if we integrate the right-hand side of Equation 3 from time \( t \) to \( t+\tau \), the resultant quantity must be zero due to the temporal periodicity.

This suggests a two-tier strategy for computing \( c \) in the unsteady case. Initially, we determine \( c \) such that the right hand side of Equation 3 is identically zero at each time step. This will permit a relatively coarse but quick determination of \( c \) and \( \Theta \). Subsequently, once \( \tau \) is well established, we use this value of \( c \) to advance \( \Theta \) for one or more periods, and monitor the decay or growth of \( \int \Theta^2 \, dV \). At the end of each trial period, we adjust \( c \) until convergence is attained.

The cell and time averaged Nusselt number based upon the center point temperature \( T_c \) at \( (x,y,z)=(b/2,H/2,W/2) \), is computed as:

\[ Nu_e = \frac{D_h}{t_f - t_i} \int_0^1 \left( \frac{(\nabla \cdot \mathbf{n})^2}{A} \right) \, ds \]  

(4)

where \( s \) includes all wall surfaces and the projected area is \( A_p = W f ds / \sqrt{2} \). The times \( t_i \) and \( t_f \) are chosen after initial transients have died out and over a sufficient number of periods that a representative temporal average is obtained. The bulk Nusselt number \( Nu_b \) is calculated by replacing \( T_c \) in the above expression by the bulk temperature defined as:
For higher Reynolds number flows in three-dimensional simulations, plots of time averaged temperature fields reveal that $T_e$ can vary by several percent in the z-direction. Thus, the choice of the spanwise location of the "probe" can have a significant impact on the value of $N_{eq}$ unless very long time averages are taken.

The three-dimensional simulations are performed in parallel, using 32-bit arithmetic, on one-hundred-twenty-eight 1860 processors on the Intel Delta at Caltech (Fischer and Patera 1991, Fischer and Ronquist 1994). The two-dimensional simulations are performed on thirty-two processors. A typical three-dimensional simulation requires fifteen seconds per time step ($\Delta t = 0.0002$ sec) for a spectral order, $N = 8$. This results in about twenty-one hours of CPU time to calculate one second of real flow time. Some Reynolds numbers can require as much as ten seconds of flow time to reach steady state conditions.

RESULTS

Velocity and Temperature Fields

A series of simulations were performed to examine the flow and heat transfer behavior of the double grooved channel for a range of Reynolds numbers. Two-dimensional simulations were performed for the fluid body force range $f_x = 0.2$ to 6.0 N/kg, while three-dimensional simulations were performed for $f_x = 0.5$ to 6.0 N/kg.

Figure 2 shows streamlines from two-dimensional simulations at $f_x = 0.2$ and 0.5 N/kg, after steady state conditions have been reached. The flow in these figures is from left to right. For $f_x = 0.2$ N/kg the flow field is time-independent. The grooves are filled with slowly turning vortices and the open channel fluid flows parallel to the x-direction. Integration of the velocity shows that the Reynolds number at this forcing level is $Re = 180$.

Figure 2b shows streamlines at six equally spaced times for a fluid forcing level of $f_x = 0.5$ N/kg. This flow exhibits a traveling wave structure, with a wavelength equal to the channel periodicity length, $b$, and a period of 0.1320 sec. Similar traveling waves are observed experimentally and numerically in a single-grooved-wall channel at Reynolds numbers above Re = 350 (Greiner et al. 1991, 1996). The traveling wave in the current channel causes fluid to be alternately ejected from the top and bottom grooves.

Velocity and temperature data from the two-dimensional simulation at $f_x = 0.5$ N/kg were replicated in the z-direction and used as initial conditions for three-dimensional simulations with the same forcing level. Figure 3a shows an isometric view of the resulting v-velocity component at the midplane $y = H/2$ after steady state conditions are reached. Figures 3b and 3c show similar snapshots for $f_x = 1.0$ and 3.0 N/kg. In these figures, fluid moves upward at x- and z-locations where the curved surface lies above the flat grid. The purpose of these plots is to show how "structure" of the flow field changes with increasing flow rate. However, the velocity scale in each of these figures is substantially different.

The surface for $f_x = 0.5$ N/kg (Fig. 3a) shows that this flow field exhibits one wave in the x-direction, and only slight variation in the z-direction. At $f_x = 1.0$ N/kg, the flow exhibits significant but regular variations in the z-direction. The variations in both the x- and z-direction are much less regular at $f_x = 3.0$ N/kg. The footprints of the peaks and valleys in these surfaces are an indication of the characteristic lengths of eddies within the flow. We see that as the fluid forcing level (and Reynolds number) increases, the flow structures become increasingly irregular and their characteristic dimensions decrease.

Figure 2 Streamlines from two-dimensional simulations. (a) $f_x = 0.2$ N/kg, (b) $f_x = 0.5$ N/kg.

Figure 3 Isometric views of v-velocity on the plane $y = H/2$. (a) $f_x = 0.5$ N/kg, (b) $f_x = 1.0$ N/kg, (c) $f_x = 3.0$ N/kg.
Figure 4 is a plot of the time dependent Reynolds number for \( f_x = 0.5, 1.0 \) and \( 3.0 \) N/kg. Line "a" is for \( f_x = 0.5 \) N/kg calculated from a two-dimensional simulation. The time average Reynolds number is \( Re_a = 383 \). Close examination of this data shows that the Reynolds number actually exhibits small oscillations with time. The period of this unsteadiness is 0.066 sec, and its amplitude 0.14% of the mean value. These flow rate oscillations are caused by the periodic ejection of slow moving fluid from the grooves. The period of the Reynolds number oscillations is half that of the unsteady flow field since there are two fluid ejection events (from the top and bottom grooves) during each cycle. The temporal variation of the Reynolds number calculated from a three-dimensional simulation for the same value of \( f_x \) is very similar to that from the two-dimensional run, and hence is not included in Fig. 4.

The time dependent Reynolds number calculated for \( f_x = 1.0 \) N/kg from a three-dimensional simulation is also shown in Fig. 4. The Reynolds number varies gently about a "constant" mean value of \( Re_a = 490 \). We note that two-dimensional simulations at this forcing level exhibit a traveling wave and give a time average Reynolds number of \( Re_a = 640 \). The lower flow rate predicted by three-dimensional simulations indicates that the structure exhibited in Fig. 3b results in higher levels of drag than two-dimensional waves. Returning to Fig. 4, the time dependent Reynolds number for \( f_x = 3.0 \) N/kg shows much larger amplitude variations about the mean value of \( Re_a = 850 \) than is exhibited at lower forcing levels. The waveform is also more random looking.

![Figure 4 Reynolds number versus time. (a) \( f_x = 0.5 \) N/kg, two-dimensional simulation. (b) \( f_x = 1.0 \) N/kg, three-dimensional simulation. (c) \( f_x = 3.0 \) N/kg, three-dimensional simulation.](image)

Figure 5 shows two isotherm plots on planes of constant z-coordinate at \( f_x = 0.5 \) and 3 N/kg. These snapshots are taken after steady state conditions are reached. The plot for \( f_x = 0.5 \) is from a two-dimensional simulation, while the plot for \( f_x = 3 \) N/kg is from a three-dimensional calculation. For \( f_x = 0.5 \) N/kg, the isotherms in the outer channel exhibit a wave structure, and the effect of the vortices in transporting fluid within the grooves is evident. Snapshots at subsequent times show that this wave travels in the positive \( x \)-direction. Isotherm plots from a three-dimensional calculation at this forcing level are very similar. Moreover, views at different z-locations are essentially identical, in agreement with the very small velocity variations in the z-direction observed in Fig. 3a.

![Figure 5 Instantaneous isotherm plots at \( z = 0 \). (a) \( f_x = 0.5 \) N/kg, two-dimensional simulation. (b) \( f_x = 3.0 \) N/kg, three-dimensional simulation.](image)

For \( f_x = 1.0 \) N/kg (not shown), the isotherm plots again exhibit smooth wave motion, similar to the pattern in Fig. 5a. However, plots at different z-locations are not in phase. Figure 5b shows isotherms for \( f_x = 3.0 \) N/kg. The temperature field is more irregular at this forcing level than it is at lower Reynolds numbers, and views at different z-locations are substantially different from Fig. 5b.

In Fig. 5, closely spaced isotherms at locations near the walls indicate large temperature gradients and thin boundary layers. For both plots the boundary layers are significantly thinner on the downstream (right-hand) groove surface than on the upstream side, as expected. They are especially thin near the groove opening. Furthermore, the boundary layers for \( f_x = 3.0 \) N/kg are significantly thinner than those at the lower forcing level. Comparing the upstream walls for these two plots, we see that a region with a relatively thin boundary layer forms near the groove trough for the higher level of forcing. This is caused by impingement of the groove vortex at this location.

![Figure 6](image)

Figure 6 shows the variation of local center point Nusselt number \( Nu_c \) from simulations (solid line) and experimental measurements (crosses, Wirtz et al. 1997). The simulation result is from a three-dimensional calculation with \( f_x = 3.0 \) N/kg \( (Re_a = 850) \). This data is for \( z = 0 \), but the variation with z-location is found to be small. The measurements are for \( Re_a = 800 \). While the simulations are intended to calculate heat transfer in the fully developed region of the channel, the measured results at this Reynolds number are in the developing region.

The simulations show that the heat flux is zero at the deepest point of the groove. Both the experiments and simulations indicate the Nusselt number is greater on the downstream portion of the groove than the upstream surface. This is consistent with the expected direction of the groove vortex rotation. Moreover, both sets of data exhibit very similar plateaus near the bottom of the upstream wall. This is expected from the isotherms shown in Fig. 5b.

The simulation results rise much more rapidly as the groove edges are approached at \( x/b = 0 \) and 1 than is observed experimentally. The area under the upstream peak of the calculated result is small compared to the area under the rest of the curve, but the downstream contribution is more significant.

Two reasons may account for the difference between the measured and calculated heat transfer. The first is the difference in
thermal development mentioned above. A second possible reason is that the numerical simulations impose a constant pressure gradient across the computational domain. The predicted ejection of relatively slow moving fluid from the grooves causes large variations in the flow rate. The experimental channel, on the other hand, has a long flat region whose velocity is not influenced by grooves. The inertia of that section may prevent the experimental flow rate from experiencing variations. The heat transfer in the numerically predicted flow, which is unsteady, would be expected to exhibit different heat transfer levels than a flow field with a more constant flow rate (Greiner 1991). Future calculations will explore constant flow rate conditions.

Figure 6 Local center point Nusselt number versus location. Line: three-dimensional simulations averaged over time at z = 0. Crosses: experimental measurements.

Figure 7 shows the time dependent bulk Nusselt number, averaged over the entire computational domain, from the same simulation conditions described in Fig. 4. For \( f_x = 0.5 \) N/kg (line "a") the fluid motion exhibited in Fig. 2b causes regular Nusselt number variations with the same period as the Reynolds number oscillations. Furthermore, the time dependent Nusselt number from a three-dimensional simulation at this forcing level (not shown) is basically the same. Generally speaking, the time scales of the Nusselt number variations exhibited in Fig. 7 are similar to those of the time dependent Reynolds number in Fig. 4. However, the amplitude of the Nusselt number unsteadiness is greater than that of the Reynolds number curves.

**Transport Quantities**

Figure 8 shows the Fanning friction factor versus Reynolds number from the current simulations (open symbols) and from experimental measurements (solid triangles, Wirtz et al. 1997). The open circles show results from two-dimensional simulations with spectral order \( N = 6 \), while squares are used for three-dimensional simulations with \( N = 8 \). Lines in the figure show laminar and transitional results for a flat passage with the same minimum wall to wall spacing as the grooved channel (Beavers et al. 1971).

Additional simulation results are included in Fig. 8 to show that the calculations are independent of the computational domain width, grid refinement and spectral order. The down-pointing triangle is from a simulation with \( N = 10 \). The open up-pointing triangle is for a more refined mesh, with six levels in the z-direction rather than four.

![Figure 7 Bulk Nusselt number versus time. (a) \( f_x = 0.5 \) N/kg, two-dimensional simulation. (b) \( f_x = 1.0 \) N/kg, three-dimensional simulation. (c) \( f_x = 3.0 \) N/kg, three-dimensional simulation.](image)

![Figure 8 Fanning friction factor versus Reynolds number from two-dimensional simulations (open circles), three-dimensional simulations (open squares), and from experiments (solid triangles). Lines show values for laminar and transitional flat ducts.](image)

Finally, the diamond is for a wider domain with \( W = 3.0 \) cm (rather than 2.0 cm) with six levels in the z-direction.
The friction factors calculated using two- and three-dimensional simulations are essentially identical for Reynolds numbers below Re = 380. At higher Reynolds numbers, the two-dimensional simulations predict that the friction factor continues to decrease with Reynolds number, similar to laminar flow in the flat passage. The three-dimensional simulations on the other hand predict that the friction factor rises rapidly from Re = 380 to 490, drops by 10% from Re = 490 to 720, and then gently increase from Re = 720 to 1175. This seemingly discontinuous behavior does not appear to be the result of a grid dependent solution. At these higher Reynolds numbers, the flow rate varies with time in a non-periodic manner, making it difficult to know how long a simulation must run in order to calculate a highly accurate mean value. The discontinuous friction factor behavior may be attributable to the uncertainty in the mean Reynolds number. Moreover, each of the simulations were run for roughly thirteen seconds of flow time (corresponding to more than 230 hours of computer time each), and averages taken over shorter time periods are slightly different from the values given in the graph.

At low Reynolds numbers, the grooved channel friction factors are below the flat passage values. To understand this result we note that these geometries have the same minimum wall to wall spacing, and this minimum spacing is used to calculate the average velocity and hydraulic diameter used in the Reynolds number and friction factor definitions. At low Reynolds numbers, the grooved passage flow fields are steady and the grooves essentially act to relax the no-slip boundary conditions. The onset of unsteadiness at Re = 380 causes its friction factor to increase beyond that of the flat passage.

The experimental data in the Reynolds number range 996 < Re < 10^4 show that the friction factor rises with Reynolds number and levels off for Re > 3800. The experimental and numerical friction factor data overlap in a fairly narrow Reynolds number range, 996 < Re < 1175. Within this range the numerically predicted values are roughly 20% below the measured values. This difference may be caused by unsteadiness of the computed flow rate. Future simulations will evaluate the friction factor under conditions of a steady flow rate and at higher Reynolds numbers.

Figure 9 shows the dependence of bulk Nusselt number on Reynolds number predicted from two-dimensional (open circles) and three-dimensional (open squares) simulations. Laminar flat channel and transitional tube results (open triangles) are shown using lines. Additional results indicate that the simulations are independent of the computational grid refinement (up-pointing triangles) and spectral order (down-pointing triangle).

The Nusselt numbers from the two- and three-dimensional simulations are identical for Reynolds numbers less than Re = 380, similar to the friction factor behavior. Moreover, three-dimensional mixing at Re > 380 causes the heat transfer to be greater than would be predicted from two-dimensional simulations. At low Reynolds numbers, the grooved channel Nusselt number is smaller than the flat passage value due to the thermal resistance of the slow moving fluid in the grooves. The onset of unsteady mixing causes the grooved channel heat transfer to increase beyond the flat passage value for Reynolds numbers above Re = 500.

Figure 10 compares the center point Nusselt numbers from the current three-dimensional simulation (open squares) with experimental data (solid triangles, Wirtz et al. 1997). As mentioned earlier, the experiments use the temperature at the center of the domain to characterize the fluid temperature instead of the bulk value. The calculations show that the center point Nusselt number is 20% below the bulk value at Re = 435, but the two values are nearly identical at Re = 850, where the flow is well mixed.

Figure 9 Bulk Nusselt number versus Reynolds number. Circles, two-dimensional simulations; squares, three-dimensional simulations. Lines show values for laminar and transitional flat ducts.

Figure 10 Center point Nusselt number from three-dimensional simulations (open symbols) and from experiments (solid triangles). Open symbols: squares, current simulation values; triangles, corrected for surface area that is excluded in the experiments.

At Re = 850, the numerically predicted Nusselt number is slightly above the line connecting measured values at Re = 800 and 1600. At
higher Reynolds numbers, however, the slope of the numerical Nu_ε versus Re dependence is significantly greater than the measured value. One possible reason for this difference is the unsteadiness of the calculated volume flow rate. Future calculations will employ constant flow rate conditions.

SUMMARY AND FUTURE WORK

Direct numerical simulations of fully-developed three-dimensional flow and convective heat transfer augmentation in a symmetrically-grooved passage with constant temperature walls have been performed over the Reynolds number range 180 < Re < 1175 using the spectral element technique. The evolution of the velocity and temperature fields as the Reynolds number is increased, from steady two-dimensional flow to coherent traveling wave structures to three-dimensional mixing, has been documented. The Fanning friction factor and Nusselt number predicted from the simulations at Re = 850 are within 20% of measured values. It is determined that two-dimensional simulations are only adequate to model flows in the symmetrically grooved passage for Reynolds numbers less than Re = 400.

Future work will simulate higher Reynolds number flows to expand the range where comparisons are made with experiments. Additional simulations that impose a constant flow rate through the computational domain (rather than a constant pressure gradient) will be performed to determine the effect of flow rate unsteadiness on the heat transfer and friction factor behavior.

ACKNOWLEDGMENT

National Science Foundation Grant CTS-9501502 supports this work. The work of P.F. Fischer is supported in part by NSF Grant ASC-9405403. The California Institute of Technology provides time on the Intel Delta under NSF Cooperative Agreement CCR-8809616.

REFERENCES


