TWO DIMENSIONAL SIMULATIONS OF ENHANCED HEAT TRANSFER IN AN INTERMITTENTLY GROOVED CHANNEL

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ABSTRACT

Two-dimensional Navier-Stokes simulations of heat and momentum transport in an intermittently grooved passage are performed using the spectral element technique for the Reynolds number range 600 ≤ Re ≤ 1800. The computational domain has seven contiguous transverse grooves cut symmetrically into opposite walls, followed by a flat section with the same length. Periodic inflow/outflow boundary conditions are employed. The development and decay of unsteady flow is observed in the grooved and flat sections, respectively. The axial variation of the unsteady component of velocity is compared to the local heat transfer, shear stress and pressure gradient. The results suggest that intermittently grooved passages may offer even higher heat transfer for a given pumping power than the levels observed in fully grooved passages.

INTRODUCTION

Engineering devices frequently employ enhanced heat transfer surfaces (Webb 1994). Fins are typically used to extend surface areas while offset strips are commonly used to promote thin boundary layers. In recent years, a number of configurations that increase fluid mixing by exciting flow instabilities have been considered. Transversely grooved channels (Ghaddar et al. 1986, Greiner 1991, and Roberts 1994), passages with eddy promoters (Kozlu et al. 1988, Karniadakis et al., 1988) and communicating channels (Amon et al. 1992) all contain fairly large features whose sizes are roughly half the channel wall to wall spacing. These structures are designed to excite normally damped Tollmien-Schlichting waves at moderately low Reynolds numbers.

The current authors have presented a series of articles on heat transfer augmentation in rectangular cross section passages with contiguous grooves cut into the walls. Flow visualizations in a long grooved channel downstream of a laminar flat passage show that the flow is steady at the inlet of the grooved section but two-dimensional waves develop after an initial development length (Greiner et al. 1990). Unsteadiness is first observed thirty-five hydraulic diameters downstream of the first groove at a Reynolds number of Re = 350. As the Reynolds number is increased, the onset location moves upstream and the flow behavior at a given location becomes increasingly three-dimensional. Experimental and numerical results in a passage with eddy promoters indicate that the instability that leads to unsteady flow is convective rather than absolute in nature (Schatz et al. 1991).

Heat transfer and pressure gradient measurements using air show that both the Nusselt number and friction factor are greater than the corresponding values for a flat channel with the same minimum wall to wall spacing (Greiner et al. 1991, Wirtz 2000).
et al. 1999). A significant result is that fully developed heat transfer is enhanced relative to laminar flat channel flow by as much as a factor of 4.6 at equal Reynolds numbers and by a factor of 3.5 at equal pumping powers.

Numerical simulations of convection in passages with one grooved surface and channels with two symmetrically grooved surfaces were performed using the spectral element technique for Re \( \leq 2000 \) (Greiner et al. 1998, 2000a). Those simulations employed three-dimensional computational domains that represented one periodicity cell of the contiguously grooved passages. Periodic inflow/outflow boundary conditions were used to model fully developed flow. The pressure gradient and heat transfer results were within 20% of the measured values for both geometries. Moreover, two-dimensional simulations did not give accurate results for Reynolds numbers greater than Re = 570. Specifically, two-dimensional heat transfer results were 20% or more below the three-dimensional results for Re \( \geq 1000 \) while two-dimensional friction factors were smaller by factors of more than two. This suggests that the three-dimensionality of these flows strongly affect their transport characteristics, especially drag.

Experimental measurements of heat transfer and pressure gradient in a flat passage downstream of a grooved channel were performed to determine the effect of decaying unsteadiness (Greiner et al. 1995, Huang 1998). Results were presented for Reynolds number range 1300 \( \leq \) Re \( \leq 5000 \). These measurements show that the heat transfer coefficient remained high for a substantial distance in the flat region. The pressure gradient on the other hand dropped back to the low flat passage value much more rapidly, especially for Re \( > 2500 \). As a result, the heat transfer for a given pumping power was even greater in the first five hydraulic diameters of the decay region than in the grooved passage itself.

Three-dimensional Navier-Stokes simulations in a flat passage downstream from a fully developed channel with symmetric, transverse grooves on two opposite walls were performed for 405 \( \leq \) Re \( \leq 764 \) using the spectral element technique (Greiner et al. 2000b). Two different computational domains were employed. The first represented one periodicity cell of a continuously grooved passage. Simulations in that domain used periodic inflow/outflow boundary conditions so that the results represent a fully developed grooved passage. The second domain consisted of a single groove coupled with a flat region. The outflow conditions from the fully developed simulation were used as the inlet conditions for the grooved/flat domain.

Unsteady flow that developed in the grooved region persisted several groove-lengths into the flat passage. This unsteadiness increased both local heat transfer and pressure gradient relative to steady flat passage flow. Moreover, the heat transfer for a given pumping power in a flat region up to three groove-lengths long was even greater than the high levels observed in a fully developed grooved passage. However, the numerical Nusselt number decayed more rapidly in the flat passage than was expected from measurements.

The favorable heat transfer versus pumping power performance of flat passages downstream from grooved channels suggests that intermittently grooved passages, in which flat regions separate grooved sections, may have significant advantages in engineering heat transfer devices. However, the development of unsteady flow in short grooved regions and the decay of non-fully-developed grooved channel unsteadiness in flat regions must be investigated before the design of intermittently grooved passages can be optimized.

The current work is a two-dimensional numerical investigation of heat transfer in an intermittently grooved passage for the Reynolds number range 600 \( \leq \) Re \( \leq 1800 \). The grooved portions of this passage have seven right-triangular slots cut symmetrically into opposite walls. The flat portion is also seven groove-lengths long and its wall to wall spacing is the same as minimum spacing in the grooved section. Earlier work has shown that three-dimensional simulations predict the pressure gradient in fully developed grooved passages much more accurately than two-dimensional calculations (Greiner et al. 1998, 2000a). However, the study of an intermittently grooved passage requires the use of a very large computational domain. The resources to perform three-dimensional simulations in this large domain are not available at the current time. Two-dimensional simulations afford an opportunity to learn more about this flow and provide guidance to future three-dimensional calculations.

**NOMENCLATURE**

- \( b \) Groove length.
- \( c \) Decay constant.
- \( d \) Groove depth.
- \( D_h \) Minimum hydraulic diameter, 2H.
- \( f_m \) Dimensionless momentum flux gradient
- \( f_p \) Fanning pressure gradient
- \( f_s \) Dimensionless axial shear stress.
- \( f_b \) Fluid body force per unit mass in the x-direction.
- \( H \) Minimum channel wall to wall spacing.
- \( k \) Fluid thermal conductivity, 0.0263 W/m°C.
- \( K \) Number of spectral elements.
- \( L_d \) Domain length, 14b
- \( N \) Spectral element order.
- \( N_u \) Bulk Nußelt number based on projected area.
- \( Pr \) Fluid molecular Prandtl number, 0.70.
- \( Re \) Reynolds number, \( U_m D_h / \nu \).
- \( t \) time.
- \( T \) Temperature.
- \( T_b \) Bulk temperature.
- \( u, v \) Velocity components in the x and y directions.
- \( u', v' \) Axial velocity unsteadiness.
- \( U_m \) Mean x-velocity at the minimum channel cross-section.

**Greek**

- \( \alpha \) Thermal diffusivity, 2.63 \( \times \) \( 10^{-3} \) m²/s
- \( \nu \) Fluid kinematic viscosity, 1.84 \( \times \) \( 10^{-2} \) m²/s.
\[ \theta \] Periodic Temperature.
\[ \rho \] Fluid Density, 1.006 kg/m³.
\[ \tau \] Period of local time variations.
\[ \tau_x \] \( x \)-component of wall shear stress.
\[ \Omega \] Computation domain

**Numerical Method**

**Computational Domain**

Figure 1 shows the two-dimensional mesh employed in this work. The upper and lower boundaries are solid walls, and the flow is from left to right (in the positive \( x \)-direction). The domain consists of seven grooves, each of length \( b = 0.024 \text{ m} \) and depth \( d = 0.012 \text{ m} \), followed by a flat section of length \( 7b \). The total domain length is \( L_d = 14b \) and the minimum passage wall to wall spacing is \( H = 0.01 \text{ m} \). The groove length was chosen to be compatible with the wavelength of the most slowly decaying Tollmien-Schlichting waves of the outer channel flow (Ghaddar et al. 1985). Moreover, the groove and passage wall to wall dimensions are the same as the geometries studied in our earlier work on decaying unsteadiness downstream of a grooved passage (Huang 1998, Wirtz et al. 1999, and Greiner et al. 2000b). However, the current domain uses periodic inlet/outlet boundary conditions and thus models fully developed flow in an array of alternating grooved and flat channels.

In the spectral element method (Patera 1984, Maday and Patera 1989) the velocity, data and geometry are expressed as tensor-product polynomials of degree \( N \) in each of \( K \) spectral elements, corresponding to a total grid point count of roughly \( KN^2 \). Numerical convergence is achieved by increasing the spectral order \( N \). The present calculations were carried out at a base resolution of \( K=1960, N=7 \), with resolution tests for \( Re=1200 \) and \( Re=1800 \) at \( N=8 \) and \( N=9 \), respectively. The present simulations use consistent approximation spaces for velocity and pressure, with pressure represented as polynomials of degree \( N = 2 \) (Maday and Patera 1989, Fischer 1997). The momentum equations are advanced by first computing the convection term, followed by a linear Stokes solve for the viscous and pressure terms. The decoupling allows for convective Courant numbers greater than unity while maintaining second-order accuracy in time. Full details of the method can be found in (Fischer 1997).

**The Periodic Domain**

The flow is driven from left to right in the periodic domain by a time-varying body force per unit mass \( f \). This forcing is determined so that the mass flow rate through the domain is invariant with time (Fischer and Patera 1992). The thermal problem for the periodic domain requires careful treatment. If one simply specifies zero-temperature conditions on the walls then the solution eventually decays to zero. To produce the desired spatially fully-developed state requires that the temperature profiles at the inlet and outlet be self-similar, that is,

\[ T(x=L_d,y,z,t) = C T(x=0,y,z,t) \]

with \( T \geq 0 \) and \( C < 1 \). The solution technique for computing the fully developed temperature field for constant temperature boundary conditions follows the analysis of Patankar et al. (1977). The energy equation, and associated initial and boundary conditions are

\[ \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{U} = \alpha \cdot \nabla^2 T \tag{1a} \]

\[ T(x,y,z,t=0) = T_{init}(x,y,z) \tag{1b} \]

\[ T(x,y,z,t) = 0 \text{ on the walls} \tag{1c} \]

\[ T(x=L_d,y,z,t) = e^{-L_d \cdot x} T(x=0,y,z,t) \tag{1d} \]

Equation 1d corresponds to the fully developed condition where the temperature profile is self-similar in each successive domain in the periodic sequence, i.e., \( T(x+L_d,y,z,t) = e^{-L_d \cdot x} T(x,y,z,t) \) for all \((x,y,z,t)\), where \( e^{-L_d \cdot x} = C \). The constant \( c \) is unknown and is a parameter to be determined as part of the computation. The fact that each domain independently satisfies the homogeneous equation (1) and that we are considering fully developed solutions that are independent of \( T_{init} \) implies that the solution to (1) for each domain would yield the same value of \( C \). Hence, \( c \) cannot be a function of \( x \). Moreover, it is readily demonstrated from energy arguments that, under fully developed conditions, \( c \) cannot be a function of time even when the flow is itself unsteady.

Any function satisfying the above self-similar condition has the unique decomposition \( T(x,y,z,t) = e^{-\eta(x,y,z,t)} \theta(x,y,z,t) \), where \( \theta(x+L_d,y,z,t) = \theta(x,y,z,t) \) is a periodic function. Thus, the computation of \( T \) is reduced to the computation of the periodic function \( \theta \), and the constant \( c \). Substituting this decomposition into equation 1 yields:

\[ \frac{\partial \theta}{\partial t} + \mathbf{U} \cdot \nabla \theta - \alpha \cdot \nabla^2 \theta - (\alpha \cdot c^2 + u_\infty) \theta - 2 \alpha \cdot \nabla \cdot (\rho \cdot \nabla \theta) \tag{2a} \]

\[ \theta(x,y,z,t=0) = \theta_{init}(x,y,z) \tag{2b} \]

\[ \theta(x,y,z,t) = 0 \text{ on the walls} \tag{2c} \]

\[ \theta(x=L_d,y,z,t) = \theta(x=0,y,z,t) \tag{2d} \]

Since the fully developed solution is independent of the initial condition we may arbitrarily assign \( \theta_{init} \) which is typically set to unity when starting from rest, or to a prior converged result when starting from an existing flow-field. Equation 2a is solved using a semi-implicit time-stepping
procedure similar to that for our Navier-Stokes solver. The
diffusive terms are treated implicitly while the convective terms
are treated explicitly. In addition, all terms on the right of
Equation 2a are treated explicitly using the latest available
value for c.

In the steady state case (∂c/∂t = 0), Equation 2 constitutes an
eigenproblem for the eigenpair (c, θ). The constant c
corresponds to the decay rate of the mean temperature in the x-
direction. As such, a larger value of c implies more rapid decay
and more effective heat transfer. In the convection-dominated
limit where the Peclet number UDu/α is large, Equation 2a
becomes a linear eigenvalue problem. In this case, standard
iterative methods for computing the lowest value of c
(corresponding to the most slowly decaying mode in x) can be
used even when the nonlinear (c²) term in Equation 2a is not
identically zero. We find that this method accurately computes
the decay rate and Nusselt numbers for steady flows in square
and round ducts (Kays and Crawford 1993).

For steady-periodic flows with period τ, the temperature is
periodic in time, implying T(x,y,z,t+τ) = T(x,y,z,t). Since c
is independent of time, this implies that θ(x,y,z,t+τ) = θ(x,y,z,t).
If the value of c is not chosen correctly, this condition will not
be satisfied. Unfortunately, τ is not known a priori but is a
result of the hydrodynamic part of the calculation. A robust
approach to computing c is obtained by multiplying Equation 2a
by θ, integrating over the domain Ω, and simplifying to yield:

\[
\frac{1}{2} \frac{d}{dt} \int \nabla \theta \cdot \nabla \theta dV = \int \left[ \nabla \cdot \left( \nabla \theta \right) (\alpha + u) \right] \theta^2 - \alpha \nabla \theta \cdot \nabla \theta dV
\]  

(3)

While we do not expect the time derivative of the average
temperature (represented by the left-hand side of Equation 3) to
be identically zero, it will in general be less than the time
derivative of θ at any one point in the domain. Moreover, if we
integrate the right-hand side of Equation 3 from time t to t+τ,
the resultant quantity must be zero due to the temporal
periodicity.

This suggests a two-tier strategy for computing c in the
unsteady case. Initially, we determine c such that the right hand
side of Equation 3 is identically zero at each time step. This
permits a relatively coarse but quick determination of c and θ.
Once τ is well established, we use this value of c to advance θ
for one or more periods, and monitor the decay or growth of θ²
dV. At the end of each trial period, we adjust c until
convergence is attained. Typical values of cμ over the range
of Re considered are 0.55 to 1.0, corresponding to 55 to 63% drops
in mean temperature over the domain length.

The time averaged bulk Nusselt number is defined as:

\[
Nu(x) = \frac{\overline{\frac{\nabla T - \overline{R}}{\overline{U}}}}{\overline{\frac{t}{u}}}
\]  

(4)

where s includes all wall surfaces. The projected wall surface
area is A_p = W/l, ds/√2. The times t_0 and t_1 are chosen after
initial transients have died out and over a sufficient number of
periods that a representative temporal average is obtained
(typically t_1 - t_0 > 0.4 sec). The bulk temperature is defined as:

\[
T_b = \frac{\int u \cdot T dV}{\int u dV}
\]  

(5)

The simulations at Re = 600 were initialized using u = 0
and θ = 1. Subsequent cases were initialized from converged
results at lower Reynolds numbers. Because of the extreme
length of the domain (14b, versus b for our earlier computations
[Greiner et al. 2000a]), very long time integrations were
required to reach a quasi-steady-periodic state at the higher
Reynolds numbers. For example, the Re=1800 case was run for a
physical time of 2.9 sec (starting from the Re = 1200 final
solution) – corresponding to roughly 9.6 convective passages
through the domain, based on mean flow-rate and domain
length. The second half of the simulation was performed with
N = 9, with a time step size of 0.000016 sec, corresponding to a
convective Courant number of 3.0. Solution files were
extracted every 100 time steps, and a total of 580 such files
were used to compute the time-averaged and rms data. This
sampling rate corresponds to roughly 50 samples per oscillation
in the solution signal.

The simulations were performed on P = 8, 16, and 32
processors of a 96 processor SGI Origin2000. Each processor
is a MIPS R10000 running at 250 MHz and shares 24 GB of
memory. The Re=1800, N=9 computation required 2.5 CPU
sec/step on 32 nodes.

RESULTS

Figure 2 shows three contour plots of the dimensionless
periodic temperature θ. These plots are typical snapshots at
Reynolds numbers Re = 600, 1200 and 1800. In this work the
Reynolds number is Re = U_m D_u/c, where the average velocity
through the minimum channel cross section is U_m =
\( (\rho u dA)/(14bH) \), the minimum channel hydraulic diameter is \( D_h = 2H \), and \( v \) is the fluid kinematic viscosity. For \( Re = 600 \), streamline plots (not shown) indicate that the central portion of the passage has essentially no transverse motion and the grooves contain slowly turning vortices that transport fluid near the downstream lip of each groove into the groove.

In Fig. 2, the temperature contours lines for \( Re = 600 \) are virtually parallel to the \( x \)-axis in the open portion of the passage and the effect of the vortices in the grooves is evident. While two-dimensional waves first appear in long continuously grooved channels at \( Re = 350 \) (Greiner et al. 1990, 1998 and 2000a), no significant wave structure is observed in the current intermittently grooved geometry at \( Re = 600 \). This indicates that the development length for unsteady flow at \( Re = 600 \) is longer than the groove section length.

At \( Re = 1200 \) a wavy structure develops in the third groove and its amplitude grows in the \( x \)-direction. This transverse motion persists for the remainder of the grooved section and for several groove-lengths into the flat region. At \( Re = 1800 \), the transverse motion is stronger and more irregular than it is at \( Re = 1200 \). It develops more rapidly in the grooved section and decays more slowly in the flat region.

Figure 3 shows contours plots of the root-mean-squared deviation of the dimensionless periodic temperature. These plots are typical snapshots at \( Re = 600, 1200 \) and 1800. While the isotherms for \( Re = 600 \) in Fig. 2 are nearly parallel to the \( x \)-axis, Fig. 3 shows that some unsteadiness develops in the fifth groove and persists roughly three groove lengths into the flat region. This unsteadiness is concentrated in the region across the groove opening. However, it does not penetrate deeply into the grooves or completely across the open channel region. The contour plots for \( Re = 1200 \) and 1800 show that as the Reynolds number is increased, unsteadiness appears nearer the entrance of the grooved section and it persists further into the flat portion. Moreover, the flow exhibits high levels of unsteadiness deep in the grooves and across the outer passage.

For \( Re = 1800 \) a significant level of unsteadiness is present at the end of the flat section (entrance to the groove region).

Figure 4 shows dimensionless axial velocity unsteadiness \( u'/U_m \) versus location and Reynolds numbers. This unsteadiness is defined as \( u'/U_m = (1/D_h)(u_{rms}/U_m)dy \), where \( u_{rms} \) is the local root-mean-squared deviation of the axial velocity from its time mean value, and the integration is taken from the bottom to the top of the channel. The region \( 0 \leq x/b \leq 7 \) corresponds to the grooved portion of the domain, while \( 7 \leq x/b \leq 14 \) represents the flat section.

At \( Re = 600 \), the velocity unsteadiness reaches maximum values of less than 2\% near the end of the grooved section. For \( Re = 1200 \), the unsteadiness grows in the first four grooves \((x/b = 0 \text{ to } 4)\), drops off slightly in the next groove \((x/b = 4 \text{ to } 5)\), and then increases for the remainder of the grooved section. The unsteadiness remains near the high values observed in the grooved section for the first half-groove-length of the flat region. It drops off very rapidly for the next two groove-lengths and then decreases at a much less rapid rate. For \( Re = 1800 \), the unsteadiness grows rapidly in the first three grooves, drops off in the fourth groove, grows again in the next two grooves and then drops off slightly in the last groove. Once again the unsteadiness remains high in the first half-groove-length of the flat region before dropping off. We do not currently understand the cause of the local dips in the unsteady velocity in the interior grooves for \( Re = 1200 \) and 1800.

Figure 5 shows bulk Nusselt number versus axial location for \( Re = 600, 1200 \) and 1800. A dashed horizontal line in the region \( 7 \leq x/b \leq 14 \) shows the heat transfer in a fully developed flat passage. The thicker line shows results at \( Re = 640 \) from a three-dimensional simulation of a fully developed grooved channel with a flat passage downstream (Greiner et al. 2000b). These results are shown in the domain \( 5 \leq x/b \leq 12 \). The bulk Nusselt number is defined as \( Nu_b = \frac{q^*}{k} \left( \frac{dT}{dx} \right)^{1/2} \), where \( q^* \) is the heat flux per unit length, \( T \) is the wall temperature, \( \Delta T \) is the temperature difference between the surface and bulk fluid, \( k \) is the fluid thermal conductivity, and \( \Delta T_b \) is the local temperature difference between the surface and bulk fluid. The heat transfer to the fluid per unit \( \text{projected surface area} \) is \( q^* = -k(\Delta T_b)/m \), where \( T \) is time average temperature, \( n \) is the direction normal to the wall and \( m \) is the wall surface direction cosine. The direction cosine in the flat region is \( m = 1 \), while it is \( m = 0.7071 \) in the grooved region. The strong singularities at
x/b = 0, 1, 2, 3, 4, 5, 6 and 7 are caused by the sharp edges of the groove peaks. Future work will consider the effect of using finite radius of curvatures at the peaks and valleys of the grooves.

The heat transfer in the first groove is very similar for all three Reynolds numbers. We see that the Nusselt number on the downstream (windward) surface (0.5 ≤ x/b ≤ 1) is significantly higher than that on the upstream (leeward) side (0 ≤ x/b ≤ 0.5). This is due to the direction of the groove vortex flow. The upstream surface exhibits a local plateau centered at x/b = 0.3. The groove vortex impinges against the wall at that location and thus the thermal boundary layer.

For Re = 600, the shape of the Nusselt number profile is similar in all seven grooves. At Re = 1200 the profile shape in the second groove is similar to that in the first, but its shape changes substantially in subsequent grooves and its magnitude increases. The heat transfer on the windward side of each groove is greater than the level exhibited at Re = 600. Moreover, the groove vortex impingement plateau grows stronger in the third through fifth groove. Its shape is essentially the same in the fifth, sixth and seventh grooves, indicating that the heat transfer has approached its fully developed condition. The profile shape has a number of local peaks especially on the leeward face. This suggests the time-averaged flow field has small secondary vortices that impinge against the walls at the locations of the peaks. For Re = 1800, the Nusselt number profile in the second groove is substantially different from its shape in the first groove. Moreover, its shape does not appear to stabilize until the sixth or seventh groove, although it is difficult to say whether it would continue to change if the grooved section were longer.

The heat transfer at the inlet of the flat region (x/b = 7) is well above the fully developed flat passage value for all three Reynolds numbers. Very steep velocity and temperature gradients near the walls cause this. The heat transfer for Re = 600 returns to the flat passage value after three groove lengths as its wall gradients approach the fully developed levels. Both

The three-dimensional results for Re = 640 (thicker line, Greiner et al. 2000b) give heat transfer levels that are substantially higher than the current Re = 600 data. As mentioned earlier, the three-dimensional results are for a fully developed grooved channel and a flat passage downstream. The flow in a fully developed grooved channel is unsteady for Re ≥ 350, while the current intermittently grooved passage is essentially steady at Re = 600. The unsteady structure of the three-dimensional simulation increases the heat transfer level well beyond that predicted by the current work for Re = 600. In fact, its level is closer to the current Re = 1200 results. Moreover, the three-dimensional simulations do not predict the secondary vortices predicted by the current two-dimensional simulations or the inflection point at x/b = 7.5.

Figure 6 shows the bulk Nusselt number averaged over different groove-length regions of the domain. A thicker solid line shows the fully developed flat passage Nusselt number. The horizontal line segments with solid squares represent fully developed results from two-dimensional simulations at Re = 600, 1200 and 1800 (Greiner et al. 2000a). Those simulations employed a computation domain that represents one periodicity cell of the grooved channel and periodic inlet/outlet boundary conditions.

For Re = 600 the average Nusselt number increases slightly for the first four grooves and then reaches a fully developed value. This value is below the flat passage level. The thermal resistance of the slowly turning groove-vortices causes this
reduction in heat transfer. For \( Re = 1200 \), the average heat transfer in the first groove is the same as that at \( Re = 600 \), but its value increases substantially in the second, third and fourth grooves. After reaching a local maximum the heat transfer drops slightly in the fifth notch and then rises slightly in the sixth and seventh grooves. However, the heat transfer level in the last four groove-lengths is fairly uniform. Moreover, this level is greater than the value for a fully developed flat passage. The unsteady mixing at \( Re = 1200 \) is sufficient to overcome the thermal resistance of the groove-vortex.

At \( Re = 1800 \) the heat transfer in the first groove is slightly higher than the level exhibited at \( Re = 600 \) and 1200. This may be caused by the unsteadiness that is present at the inlet to the grooved section (Figs. 3 and 4). The heat transfer exhibits a substantial rise in next two grooves. It drops in the forth groove and rises for the next two grooves (similar to the behavior in grooves 5, 6 and 7 of at \( Re = 1200 \)), before dropping slightly in the final groove. The average Nusselt numbers in all but the first groove is greater than the fully developed flat passage value. The rise and fall of heat transfer in the interior grooves at \( Re = 1200 \) and 1800 is closely correlated with the unsteady velocity levels described in Fig. 3.

At \( Re = 600 \), the heat transfer in the seventh groove is substantially less than the level predicted for fully developed flow. The unsteadiness that develops in the fully developed simulation causes this difference. At \( Re = 1200 \) and 1800 the average levels heat transfer in the seventh groove are, respectively, 2% and 7% lower than the fully developed values. The local heat transfer data presented in Fig. 5 indicate that the profile for \( Re = 1200 \) is closer to being fully developed at the end of the grooved section than is the profile for \( Re = 1800 \). This may explain the larger difference between the last groove of the current simulation and the fully developed result for \( Re = 1800 \).

In the flat region (\( 7 \leq x/b \leq 14 \)), the heat transfer at all three Reynolds numbers begins at very high levels. It decreases as distance from the grooves increases, eventually approaching the fully developed value. It appears that the initial development length for heat transfer in the groove region (as demonstrated by a rise in heat transfer) decreases with Reynolds number. Moreover, the heat transfer decay-length in the flat region increases with Reynolds number.

Figure 7 shows the dimensionless x-component of shear stress \( f_x \) versus axial location and Reynolds number. This shear stress is defined as \( f_x = 2 \tau_x / \rho U_m^2 \). In this expression the x-component of wall shear stress is \( \tau_x = -\mu (ds/dn) \), where \( s \) is time average fluid speed, \( \rho \) is the fluid density and \( \mu \) is the fluid dynamic viscosity. The strong negative singularities in Fig. 7 are caused by the sharp groove peaks at \( x/b = 0, 1, 2, 3, 4, 5, 6 \) and 7.

For \( Re = 600 \), the shape of the shear stress profile in the grooves (\( 0 \leq x/b \leq 7 \)) has a number of similarities to the Nusselt number profiles seen in Fig. 5. For instance, the shear stress is significantly higher on the downstream surface of each groove

![Figure 7 Dimensionless axial shear stress versus location and Reynolds number.](image)

than it is on the upstream side. Moreover, the shear stress exhibits a plateau at 0.3 groove-lengths downstream from the leading edge of each groove. These similarities are caused by the analogous behavior of heat and momentum transport in the absence of strong pressure gradients.

At \( Re = 600 \) the shear stress is positive throughout the grooved region (with the exception of the singular points). This indicates that the fluid near the groove surfaces is always moving in the negative x-direction. For \( Re = 1200 \) and 1800, on the other hand, the shear stress is negative in certain locations. This implies that fluid in these regions moves in the positive x-direction and supports the theorized existence of secondary vortices described in connection the jagged peaks in Fig. 5.

In the flat region downstream from the grooves, the dimensionless shear stress is in the negative x-direction. Figure 7 shows that the magnitude of the shear stress in the flat region is much higher than that in the grooved region.

Comparing Figs. 5 and 7 shows that the wall shear stress is analogous to heat transfer. However, the pressure gradient is the drag characteristic of importance for engineering devices since it affects the pumping power consumed by a prime-mover for a given flow rate. We now relate the Fanning pressure gradient \( f_p \) to the wall shear stress. The Fanning factor is defined as \( f_p = (-dF_p/dx)/[(1/2pU_m^2)] \). In this expression the pressure force is \( F_p = \int p dy \), where \( p \) is the local pressure and the integration is performed from the top to the bottom of the channel. A time-averaged force balance on a control volume of axial length \( c \) shows that the Fanning pressure gradient is composed of wall shear stress and momentum flux gradient components, that is \( f_p = f_m - f_s \). The axial gradient of the momentum flux is defined as \( f_m = d/dx[(u^2)/U_m^2]dy \), where \( u^2 \) is the time average of the square of the x-velocity, and the integration is performed from the bottom to the top of the passage.

Figure 8 shows the axial gradient of the momentum flux \( f_m \) versus location and Reynolds number. In the first groove, the momentum flux gradient is similar for all three Reynolds
numbers. In subsequent grooves, the magnitude of parameter increases with Reynolds numbers. This indicates that the fluid experiences large levels of acceleration and deceleration. In the flat region downstream from the grooves, the axial variation of the velocity is small and $f_M$ drops to zero very quickly. Comparing Figs. 7 and 8 (and noting the y-axis scales), we see that the magnitude of the momentum flux gradient $f_M$ is much larger than that of the dimensionless shear stress in the grooved regions, while the opposite is true in the flat portion of the passage. The local Fanning pressure gradient is the difference between the traces in these figures, $f_p = f_M - f_s$.

Figure 9 shows the Fanning pressure gradient averaged over different groove-length regions. This dependence is shown as a function of axial location and Reynolds numbers. The thinner horizontal lines in the region $7 < x/b < 14$ show the fully developed Fanning factor for flat channels, $f_p = 24/Re$. In the first groove the Fanning pressure gradient is roughly equal for all three Reynolds numbers. Moreover, it is negative for this region indicating that the dimensional pressure actually increases across this region. This pressure rise is caused by fluid deceleration. The expansion of the channel cross section as the flow exits a flat passage and enters the grooved region causes this deceleration. For $Re = 600$, the friction factor becomes positive in the second groove and continues to increase with distance from the inlet of the grooved section. The increment in the groove averaged Fanning factor decreases with increasing $x/b$ until the last groove. The pressure decrease across the last groove is large because the flow accelerates as it exits that groove and enters the flat region.

For $Re = 1200$, the Fanning factor increases for the first five grooves, then decreases in the next groove, and finally increases in the last groove. The pressure gradient for $Re = 1800$ increases for the first four grooves, decreases in the next groove, and rises in the final two grooves. We see that the pressure gradient development length in the grooved section increases with Reynolds number.

In the flat region, the pressure gradients for all three Reynolds numbers are very high in the first groove length due to the high shear stress (sharp velocity gradients) at $x/b = 7$.

Figure 9 Groove-length average Fanning pressure gradient versus location and Reynolds number.

The pressure gradients approach their respective fully developed values after only three groove lengths in the flat passage. The pressure gradient is below the fully developed flat passage value in the last groove length of the flat region. This is due to the decelerating flow caused by the expanding cross sectional area at $x/b = 14$.

Comparing Figs. 6 and 9, we see that while the heat transfer remains enhanced for up to six groove-lengths in the flat region, the pressure gradient drops back to the flat passage value in only three groove lengths. Moreover, while the decay length for heat transfer increases with Reynolds numbers, the pressure gradient decay length is rather insensitive to Reynolds number.

CONCLUSIONS

Two-dimensional Navier-Stokes simulations of heat and momentum transport in an intermittently grooved passage were performed using the spectral element technique for the Reynolds number range $600 \leq Re \leq 1800$. The computational domain had seven contiguous transverse grooves cut symmetrically into opposite walls, followed by a flat section of the same length. This domain employed periodic inflow/outflow boundary conditions.

The flow is essentially steady at $Re = 600$. However, traveling waves develop near the inlet of the grooved section at $Re = 1200$ and $1800$ and persist several groove lengths into the flat region. The axial variation of unsteady velocity within the grooved section is closely correlated with increases in heat transfer, shear stress and pressure gradient. The development length for heat transfer in the grooved region decreases with Reynolds number, while the length of the flat region where heat transfer augmentation is observed increases with Reynolds number. The pressure gradient development length in the grooved region increases with Reynolds number. However, in the flat section, the gradient returns to the fully developed value in roughly three groove lengths for the entire Reynolds number range considered in this work. These results suggest that intermittently grooved passages may offer very favorable heat
transfer versus pumping power performance for engineering devices.

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