Solutions to homework no. 3

§3-E28 Apply proposition 3.18 to each of the three pair of angles $\{\angle A, \angle B\}$, $\{\angle A, \angle C\}$ and $\{\angle B, \angle C\}$.

§3-E34 (a) Let $A = (0,0)$, $B = (1,1)$ and $C = (2,0)$ and pick the ray $\vec{r}$ to be $\vec{AC}$. By $\mathcal{C}.\mathcal{A}.1$ there exists a (unique) point B' on $\vec{r}$ such that $AB \cong AB'$. By necessity the point B' has to be $B' = (\sqrt{2}, 0)$ which however is a nonexistent point in $\mathbb{Q}^2$.

(b) Let γ be the circle centered at A and with radius AB. Then the segment AC has endpoints inside and outside of γ but the intersection point of γ with AC would again be B' which is not in $\mathbb{Q}^2$.

§3-E36 The gist of this problem is to use the fact (belief!) that the axioms of neutral geometry hold true in the Euclidean plane and the circle γ under consideration lies in that plane.

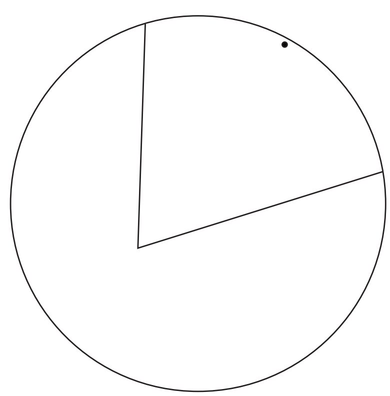
$\mathcal{B}.\mathcal{A}.1$ Let A, B, C be three distinct points inside of γ. The definition of betweenness (which is the same inside of γ as it is in the Euclidean plane) shows that A, B and C lie on a line (and hence on a chord) and that $A * B * C$ and $C * B * A$ are simultaneously true.

$\mathcal{B}.\mathcal{A}.2$ Follows from $\mathcal{B}.\mathcal{A}.2$ in Euclidean geometry and the fact that the points on γ are not part of the model.

$\mathcal{B}.\mathcal{A}.3$ Follows from $\mathcal{B}.\mathcal{A}.3$ in Euclidean geometry.

(a)]$\mathcal{B}.\mathcal{A}.4$] Again just follows from $\mathcal{B}.\mathcal{A}.4$ in Euclidean geometry.

For the second half of the problem see the picture below.



Prop. 1.13a Given the segment AB and the ray $\vec{CD}$ there exists by $\mathcal{C}.\mathcal{A}.1$ a unique point $E \neq C$ on $\vec{CD}$ such that $AB \cong CE$. Then either

$E = D$ or by $\mathcal{B}.\mathcal{A}.3$ either $C * E * D$ or $C * D * E$. These three possibilities are mutually exclusive.

- If $E = D$ then $AB \cong CD$.
- If $C * E * D$ the by definition $AB < CD$.
- If $C * D * E$ then proposition 3.12 ensures that there is a point F between A and B such that $CD \cong AF$. But then (again by definition) $AB > CD$.

Prop. 1.13b By $AB < CD$ we can find a point X on CD such that $AB \cong CX$. By proposition 3.12 (since $CD \cong EF$) there exists a point Y on EF such that $CX \cong EY$. But then by definition $AB < EF$.