Chapter 9. Injections, surjections, functions and sets.

Injective is the same as **one-to-one**.

Surjective is the same as **onto**.

Bijective is the same as **one-to-one and onto**.

Example for class: Prove that $f : R \to R$ by $f(x) = 4x - 3$ is bijective. Prove that $h : N \to N$ by $h(x) = 4x - 3$ is injective but not surjective. Prove that $g : R \to R^{\geq}$ by $g(x) = x^2$ is surjective but not injective.

Exercise 29 *Let $f : X \to Y$ and $g : Y \to Z$. Prove the following:*

(a) If g and f are injective then $g \circ f$ is also.

(b) If $g \circ f$ is injective then f is also.

(c) Find a simple example where $g \circ f$ is injective but g is not injective.

(d) If g and f are surjective then $g \circ f$ is also. (Same as Eccles #18 page 118.)

(e) If $g \circ f$ is surjective then g is also.

(f) Find a simple example where $g \circ f$ is surjective but f is not surjective.

See Remarks 9.3.2 in Eccles. He uses unconventional notation. He writes $\overrightarrow{f}(S) = \{f(x)|x \in S\}$ whereas most authors simply use $f(S)$. In Sec 8.4 Eccles defines the image of a function $f : X \to Y$. Here it can be written as $f(X)$ or $\overrightarrow{f}(X)$ He also uses $\overleftarrow{f}(T) = \{x|f(x) \in T\}$ where most authors use $f^{-1}(T)$.

It is very important to distinguish the two ways in which f^{-1} is normally used. When T is a set, $f^{-1}(T)$ means the same as $\overleftarrow{f}(T)$ and has nothing to do with the inverse of a function. If $f : X \to Y$ is a bijection then f^{-1} is a function $f^{-1} : Y \to X$ called the inverse of f. It's the context that tells you how to interpret it.

Exercise 30 *In each of the following statements, assume $f : X \to Y$. If it is true prove it. If it is false, give a counter example.* ***Compare these with exercise 9.7 on Eccles page 114 and problem 20 on Eccles page 118. You should try them first as they are special cases of the ones below.***

(a1) True or false? Eccles: $\overrightarrow{f}(\bigcup_{i\in I} B_i) \subseteq \bigcup_{i\in I} \overrightarrow{f}(B_i)$. Conventional: $f(\bigcup_{i\in I} B_i) \subseteq \bigcup_{i\in I} f(B_i)$.

(a2) True or false? Eccles: $\overrightarrow{f}(\bigcup_{i\in I} B_i) \supseteq \bigcup_{i\in I} \overrightarrow{f}(B_i)$. Conventional: $f(\bigcup_{i\in I} B_i) \supseteq \bigcup_{i\in I} f(B_i)$.

(b1) True or false? Eccles: $\overrightarrow{f}(\bigcap_{i\in I} B_i) \subseteq \bigcap_{i\in I} \overrightarrow{f}(B_i)$.

(b2) True or false? Eccles: $\overrightarrow{f}(\bigcap_{i\in I} B_i) \supseteq \bigcap_{i\in I} \overrightarrow{f}(B_i)$.

(c1) True or false? Eccles: $\overleftarrow{f}(\bigcup_{i\in I} B_i) \subseteq \bigcup_{i\in I} \overleftarrow{f}(B_i)$. Conventional: $f^{-1}(\bigcup_{i\in I} B_i) \subseteq \bigcup_{i\in I} f^{-1}(B_i)$.

(c2) True or false? Eccles: $\overleftarrow{f}(\bigcup_{i\in I} B_i) \supseteq \bigcup_{i\in I} \overleftarrow{f}(B_i)$. Conventional: $f^{-1}(\bigcup_{i\in I} B_i) \supseteq \bigcup_{i\in I} f^{-1}(B_i)$.

(d1) True or false? Eccles: $\overleftarrow{f}(\bigcap_{i\in I} B_i) \subseteq \bigcap_{i\in I} \overleftarrow{f}(B_i)$.

(d2) True or false? Eccles: $\overleftarrow{f}(\bigcap_{i\in I} B_i) \supseteq \bigcap_{i\in I} \overleftarrow{f}(B_i)$.