Section 6.3

Some solids of revolution are difficult to handle with the methods developed in section 6.2.

\[ y = 2x^2 - x^3 \]

Finding the inner and outer radiiuses for the washer method of section 6.2 requires solving \( y = 2x^2 - x^3 \) for \( x \). We will return to this example after discussing the Shell method.
A typical shell, cut and flattened as in Figure 5, with radius $x$, circumference $2\pi x$, height $f(x)$, and thickness $\Delta x$ or $dx$:

$$\int_{a}^{b} (2\pi x) [f(x)] \, dx$$

Figure 5
Example 1

Find the volume of the solid obtained by rotating about the $y$-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

Solution:

From the sketch in Figure 6 we see that a typical shell has radius $x$, circumference $2\pi x$, and height $f(x) = 2x^2 - x^3$. 
Example 1 continued

\[ V = \int_0^2 \left( 2\pi x \right) \left( 2x^2 - x^3 \right) \, dx \]

The height is the circumference from \( \Delta x \) which was the thickness

\[ V = 2\pi \int_0^2 (2x^3 - x^4) \, dx \]

\[ = 2\pi \left[ \frac{1}{2} x^4 - \frac{1}{5} x^5 \right]_0^2 \]

\[ = 2\pi \left( 8 - \frac{32}{5} \right) \]

\[ = \frac{16}{5} \pi \]
ex  Find the volume of the solid obtained by revolving the triangle with vertices (2, 2), (2, 8) and (5, 8) about the line $x = 9$

The line through (2, 2) and (5, 8)

$y - 2 = \frac{8 - 2}{5 - 2}(x - 2)$

$y = 2x - 8$
Example continued

\[ \int_{2}^{5} \left( \frac{\text{height}}{8 - (2x - 2)} \right) \text{circumference} \frac{2\pi(9 - x)}{\text{radius}} \, dx \]

= 108\pi \text{ cubic units.}
Ex: Find the volume of the solid obtained by revolving the triangle with vertices \((2,2), (2,8)\), and \((5,8)\) about the line \(y = 2\).
Ex Find the volume of the solid obtained by revolving the triangle with vertices (2,2), (2,8) and (5,8) about the line y = 2.

\[ \int_{2}^{8} \left( \frac{y}{2} + 1 \right) \cdot 2 \pi (y - 2) \, dy = 72 \pi \]