Practice Problem Answers

4.3 (a) $r^2 - 10 = 0$

$r = \pm\sqrt{10}$ so $y(t) = C_1 e^{-\sqrt{10}t} + C_2 e^{\sqrt{10}t}$

corrected problem

(b) $r^3 + 3r^2 + 3r + 1 = (r+1)^3$

$y(t) = C_1 e^{-t} + C_2 t e^{-t} + C_3 t^2 e^{-t}$

(c) $r^2 - 4r + 9 = 0$

$(r - (2+\sqrt{5}i))(r - (2-\sqrt{5}i)) = 0$

$y(x) = e^{2t}[C_1 \cos(\sqrt{5}t) + C_2 \sin(\sqrt{5}t)]$

4.4 (a) $y(x) = y_c(x) + y_p(x) = c_1 e^{(-2+\sqrt{3})t} + C_2 e^{(-2-\sqrt{3})t} + 3x - 7$

(c) $y(x) = y_c(x) + y_p(x)$

$= \underline{c_1 + c_2 e^{-x} +} 2x - xe^{-x}$

$y'(x) = -c_2 e^{-x} + 2 - e^{-x} + xe^{-x}$

$10 = y(0) = c_1 + c_2$

$15 = y'(0) = -c_2 + 1 \Rightarrow c_2 = -14$

so $c_1 = 24$

$y(x) = 24 - 14e^{-x} + 2x - xe^{-x}$

Practice Problem Answers

4.2 Reduction of order $xy'' + y' = 0$ has sol'n $y_0(x) = \ln(x)$. A second sol'n is $y = uy_1$ where

$$u(x) = \int \frac{e^{-\int \frac{dx}{x}}}{(y_1)^2}\,dx = \int \frac{\frac{1}{x}}{(\ln x)^2}\,dx$$

$$= \int \frac{dv}{v^2} = \frac{1}{v} = \frac{1}{\ln x}$$

$\boxed{\begin{array}{l} v = \ln x \\ dv = \frac{dx}{x} \end{array}}$ so $\boxed{y = 1}$ is a second solution.

4.7 (a) Auxiliary equation

$$m^2 - 3m - 4 = 0$$

$$(m-4)(m+1) = 0$$

two sol'ns $y_1(x) = x^4$, $y_2(x) = \frac{1}{x}$

general sol'n $y(x) = c_1 x^4 + c_2 \frac{1}{x}$

(b) Auxiliary equation

$$4m^2 + 4m + 1 = 0$$

$$(2m+1)^2 = 0$$

$$y(x) = c_1 x^{-1/2} + c_2 x^{-1/2} \ln x$$