Exam 1
Solutions for practice problems 2-5

(2) (a) $\frac{dy}{dx} + y = x$, here $P(x) = 1$

So the I.F. is $e^{\int P dx} = e^x$.

$$\frac{d}{dx}(ye^x) = xe^x$$

$$ye^x = \int xe^x dx = xe^x - e^x + C$$

$$y = x - 1 + Ce^{-x} \quad //$$

(2) (b) $x\frac{dy}{dx} + y = x$

Std form $\frac{dy}{dx} + \frac{1}{x}y = 1$, here $P(x) = \frac{1}{x}$

So the I.F. is $e^{\int P dx} = e^{\ln x} = x$

$$\frac{d}{dx}(yx) = 1 \cdot x$$

$$yx = \frac{x^2}{2} + C, \quad y = \frac{x}{2} + \frac{C}{x}$$

(3) Newton's Law of Heating/Cooling

$$\begin{cases} \frac{dT}{dt} = k(T-300) \\ T(0) = 70 \end{cases}$$

Also, $T(1/2) = 120$

Separable variables: $\frac{dT}{T-300} = k\,dt$

$$\ln|T-300| = kt + C_1$$

$$T - 300 = e^{kt+C_1} = Ae^{kt}$$

$$\boxed{T = 300 + Ae^{kt}}$$

To solve for A, use $70 = T(0) = 300 + A$ so that $A = -230$. $T = 300 - 230e^{kt}$

To solve for k, use $120 = T(1/2) = 300 - 230e^{k/2}$

$$k = 2\ln\left(\frac{18}{23}\right)$$ //

(4) **Mixing Problem** $A(t)$ amount of salt in tank at time t.

$$\begin{cases} \frac{dA}{dt} = (2)(3) - (3)\left(\frac{A}{100-t}\right) \\ A(0) = 0 \end{cases}$$

---

(5) $$\frac{dP}{dt} = P(a-P), \quad a > 0$$

$$\frac{d^2P}{dt^2} = P(a-P)(a-2P)$$

this give the "interesting" inflection information.