① $z = f(x,y) = -6x^2 - 2y^2$

$\vec{n} = \left\langle \frac{\partial f}{\partial x}(2,1), \frac{\partial f}{\partial y}(2,1), -1 \right\rangle$ normal

$\vec{n} = \langle -12(2), -4(1), -1 \rangle$

$= \langle -24, -4, -1 \rangle$

plane: $-24(x-2) - 4(y-1) - (z+26) = 0$

(B) $\boxed{-24x - 4y - z = -26}$

② $x^2 + 5xyz + y^2 + 7z^2 = 0$

oops (1, 1, 1) Not on surface! ☹

Try $\underbrace{x^2 + 5xyz + y^2 - 7z^2}_{G(x,y,z)} = 0$

$\nabla G(x,y,z) = \langle 2x + 5yz, 5xz + 2y, 5xy - 14z \rangle$

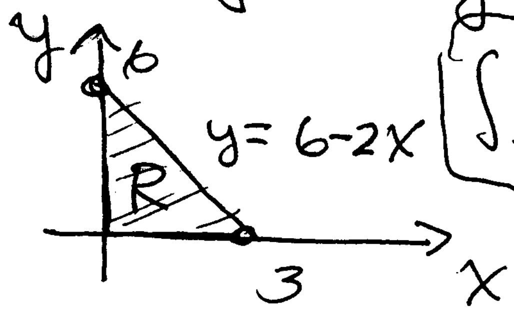
$\nabla G(1,1,1) = \langle 7, 7, -9 \rangle$

plane: $7(x-1) + 7(y-1) - 9(z-1) = 0$

$7x + 7y - 9z = 5$

Answer (A) after repairing the problem.

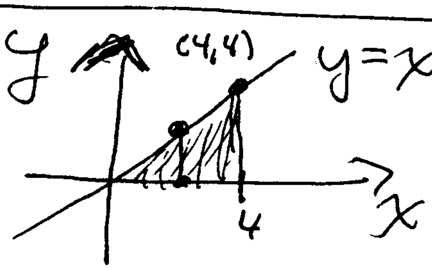
(3) $f(x,y) = xy$



$$\iint_R xy\,dA = \int_0^3\int_0^{6-2x} xy\,dy\,dx = \int_0^3 x\frac{y^2}{2}\Big|_0^{6-2x}\,dx$$

$$= 27/2 \quad (B)$$

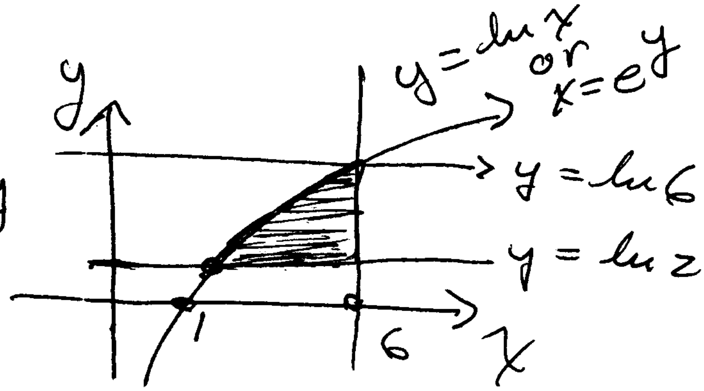
(4) $\displaystyle\int_0^4\int_0^x dy\,dx$



$$= \int_0^4\int_y^4 dx\,dy$$

Answer (C)

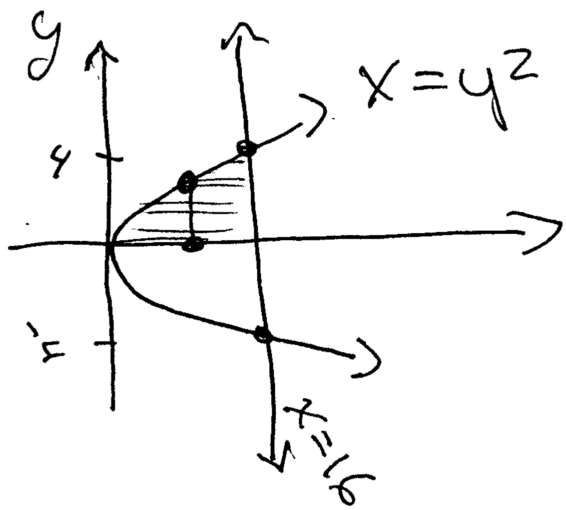
(5) $\displaystyle\int_{\ln 2}^{\ln 6}\int_{e^y}^{6} 4y\,dx\,dy$



$\displaystyle = \int_{2}^{6}\int_{\ln 2}^{\ln x} 4y\,dy\,dx$

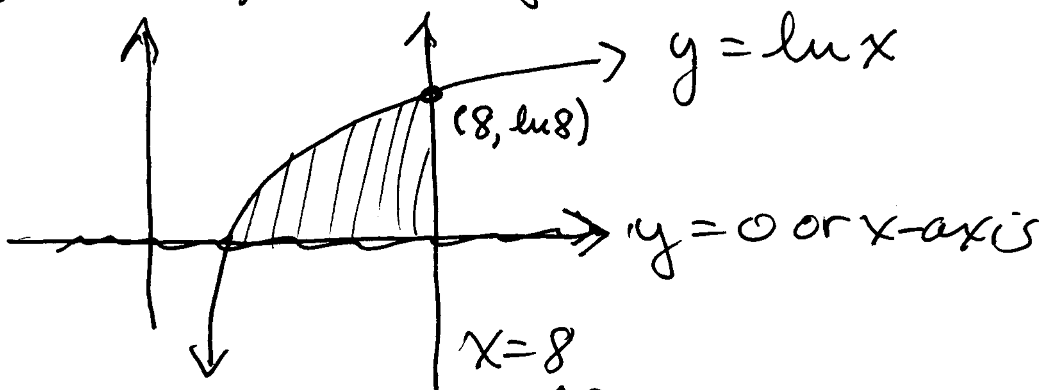
Answer (A)

(6) $\displaystyle\int_{0}^{4}\int_{y^2}^{16} 5y\,dx\,dy = \int_{0}^{16}\int_{0}^{\sqrt{x}} 5y\,dy\,dx$



answer (1)

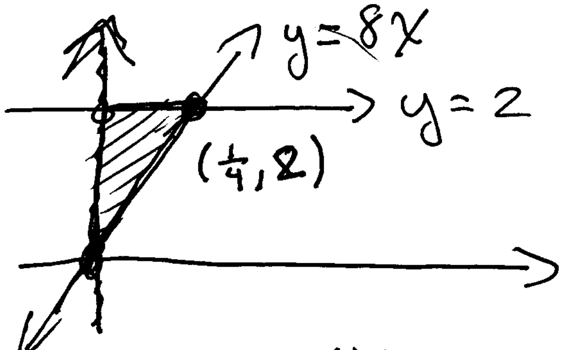
(7) $y = \ln x$, $x = 8$, $y = 0$



$$\int_1^8 \int_0^{\ln x} dy\,dx = \int_0^{\ln 8}\int_{e^y}^{8} dx\,dy$$

answer: none of these

(8)



$$\int_0^2 \int_0^{y/8} dx\,dy = \int_0^{1/4}\int_{8x}^{2} dy\,dx$$

Answer (13)

(9) $f(x,y) = 2xy + 5x - 8y$

$$\begin{cases} f_x = 2y + 5 \overset{set}{=} 0 \\ f_y = 2x - 8 \overset{set}{=} 0 \end{cases}$$

critical point $(4, -5/2)$

$$f_{xx} f_{yy} - (f_{xy})^2 = (0)(0) - (2)^2 = -4 < 0$$

Saddle pt.

---

(10) $f(x,y) = x^3 + y^3 - 48 - 243y + 7$

$$\left.\begin{aligned} f_x &= 3x^2 - 48 \overset{set}{=} 0 \\ f_y &= 3y^2 - 243 \overset{set}{=} 0 \end{aligned}\right\}$$ critical points $(4,9), (4,-9)$, $(-4,9), (-4,-9)$

Let $D(x,y) \overset{def}{=} f_{xx} f_{yy} - f_{xy}^2$
$= 36xy - 36$

at $(4,9)$: $D > 0$, $f_{xx}(4,9) > 0$ min

at $(-4,9)$: $D < 0$ Saddle

at $(4,-9)$: $D < 0$ Saddle

at $(-4,-9)$: $D > 0$, $f_{xx}(-4,-9) < 0$ max

answer B

Chain Rule

$$\frac{dw}{dt}(1) = \underbrace{F_u(u(1),v(1))}^{2u(1)}\,u'(1) + \overbrace{F_v(u(1),v(1))}^{4v(1)+1}\,v'(1)$$

$$= 2u(1)\,u'(1) + (4v(1)+1)\,v'(1)$$

$$= 2(5)(3) + (4\cdot 7+1)(2)$$

$$= 30 + 58$$

$$= 88 \quad //$$