Solutions to Ch 12 practice multiple choice -

① $f(x,y,z) = \ln(x^2 - 5y^2 + 8z^2)$

$$\nabla f(x,y,z) = \frac{1}{x^2 - 5y^2 + 8z^2} \langle 2x, -10y, 16z \rangle$$

$$\nabla f(-5,-5,-5) = \left\langle \frac{-1}{10}, \frac{1}{2}, \frac{-4}{5} \right\rangle$$

answer D

② $f(x,y) = -5x^2 + 7y$ at $P(9,-2)$ and direction $A = \langle 3, -4 \rangle$.

$$\nabla f(x,y) = \langle -10x, 7 \rangle$$

$$\nabla f(9,-2) = \langle -90, 7 \rangle$$

$$D_{\frac{A}{|A|}} f(9,-2) = \langle -90, 7 \rangle \cdot \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle$$

$$= \frac{-298}{5}$$

(3) $f(x,y) = xy^2 - yx^2$

$P_0(-1,2)$

$\nabla f(x,y) = \langle y^2 - 2xy, 2xy - x^2 \rangle$

$\nabla f(-1,2) = \langle 2^2 - 2\cdot \overset{-1 \text{ oops}}{1} \cdot 2, 2(-1)(2) - (-1)^2 \rangle$

$= \langle 8, -5 \rangle$ Answer

any multiple of $\langle 8, -5 \rangle$.

So C or D in my opinion.

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(4) $f(x,y) = x^2 + xy + y^2$

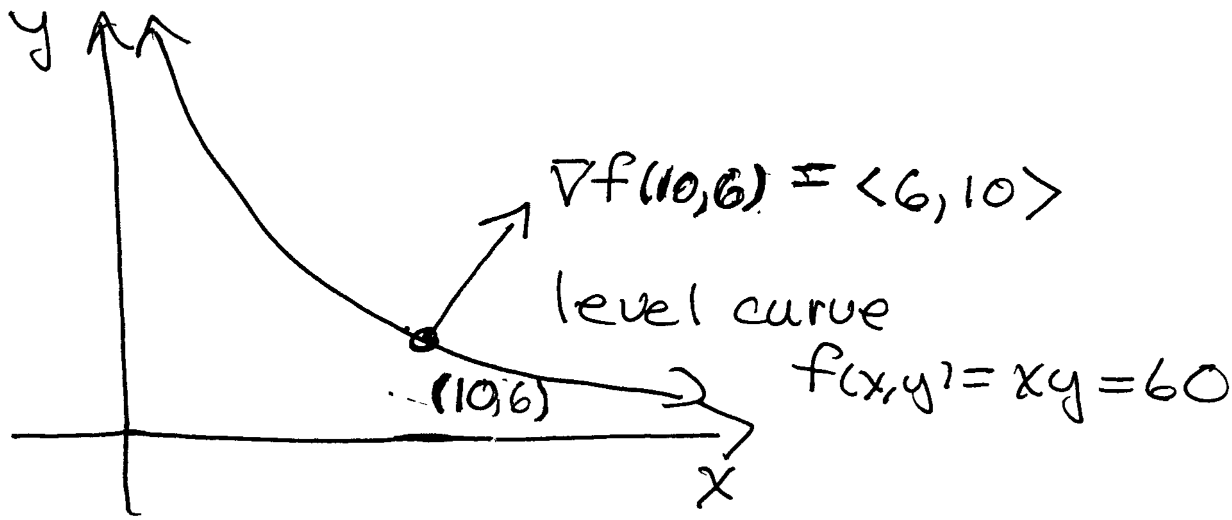
$\nabla f(x,y) = \langle 2x + y, x + 2y \rangle$

$\nabla f(-2,-1) = \langle -5, -4 \rangle$

Answer: $-|\nabla f(-2,-1)| = -\sqrt{41}$

Note: $+|\nabla f(-2,-1)|$ would give greatest rate of increase.

5



Tangent line at $(10,6)$

$$\nabla f(10,6) \cdot \langle x-10, y-6 \rangle = 0$$

$$6(x-10) + 10(y-6) = 0$$

Answer: D

6

$$F(x,y,z) = 9x - 8y + 7z \overset{set}{=} 31$$

$$\nabla F(x,y,z) = \langle 9,-8,7 \rangle$$

$$\vec{r}(t) = \langle 1,-1,2 \rangle + t\langle 9,-8,7 \rangle$$

$$= \langle 1+9t, -1-8t, 2+7t \rangle$$

answer C

(7) objective function $f(x,y) = xy$

constraint $g(x,y) = x^2+y^2 \overset{set}{=} 128$

Solve $\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) = x^2+y^2 = 128 \end{cases}$

$\begin{cases} y = \lambda 2x & (1) \\ x = \lambda 2y & (2) \\ x^2+y^2 = 128 & (3) \end{cases}$

Divide (1) into (2) $\frac{y}{x} = \frac{x}{y}$

so $y^2 = x^2$. Substitute this into (3) $2x^2 = 128$ or $x^2 = 64$

so $x = \pm 8$ and $y = \pm 8$

$f(8,8) = 64$, $f(-8,-8) = 64$

$f(-8,8) = -64$, $f(8,-8) = -64$

Answer C

8

$f(x,y,z) = x + 2y - 2z$ objective function

$g(x,y,z) = x^2+y^2+z^2 = 9$ constraint

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = 9 \end{cases}$$

solve for x,y,z

$$\begin{cases} 1 = 2\lambda x \\ 2 = 2\lambda y \\ -2 = 2\lambda z \\ x^2+y^2+z^2=9 \end{cases}$$

$$\begin{cases} x = \frac{1}{2\lambda} \\ y = \frac{1}{\lambda} \\ z = \frac{-1}{\lambda} \end{cases}$$

$$x^2+y^2+z^2 = 9 = \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 + \left(\frac{-1}{\lambda}\right)^2$$

So $\frac{9}{4}\frac{1}{\lambda^2} = 9$ or $\lambda^2 = \frac{1}{4}$ so $\boxed{\lambda = \pm\frac{1}{2}}$

$(x,y,z) = (1,2,-2)$ and $(x,y,z) = (-1,-2,2)$ are points of interest

$f(1,2,-2) = 9$ Answer C

$f(-1,-2,2) = -9$