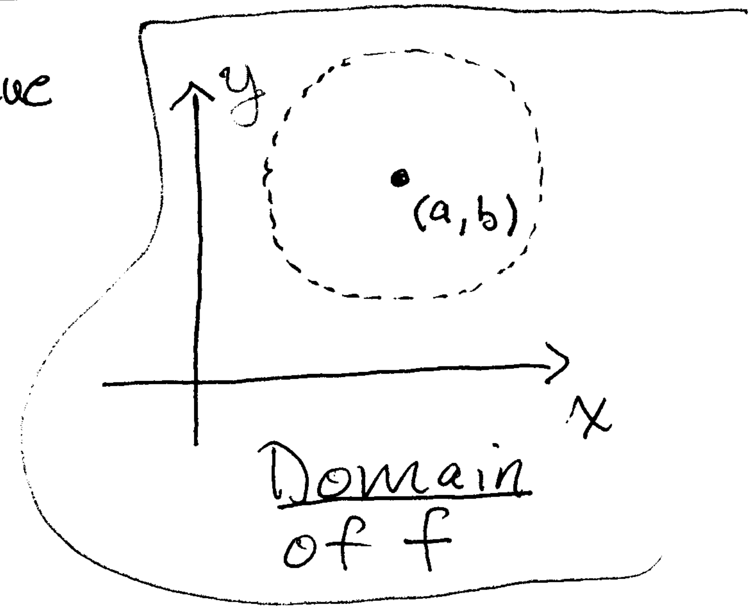
12.7 <u>Part 1</u> 1

Extrema of functions of two variables. $z = f(x,y)$.

<u>Definition</u>: <u>Local Maximum</u> $(a, b, f(a,b))$

$f(a,b)$ is a local max value if –



<u>Definition</u>: <u>Local Minimum</u>

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(From Theorem 10) First Derivative Test.

If f has a local maximum or minimum at a point (a, b) in the interior of its domain and if the first partials of f exist, then

$$f_x(a, b) = 0 \qquad f_y(a, b) = 0$$

Explanation: Think of tangent plane.

Definition: An interior point of the domain of f is called a critical point if either

a) one of f_x or f_y does not exist at the point

b) both f_x and f_y are zero at the point.

ex:

<u>Definition</u> A differentiable function f has a <u>saddle point</u> at a critical point (a, b) if <u>in</u> <u>every</u> open disk centered at (a, b) there are domain points (x_1, y_1) and (x_2, y_2) such that

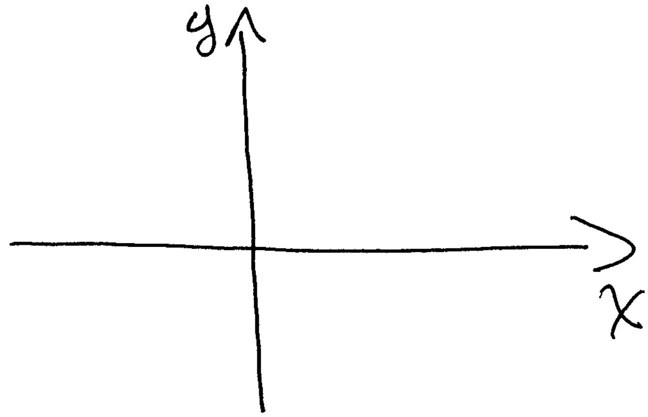
a) $f(x_1, y_1) < f(a, b)$

b) $f(a, b) < f(x_2, y_2)$

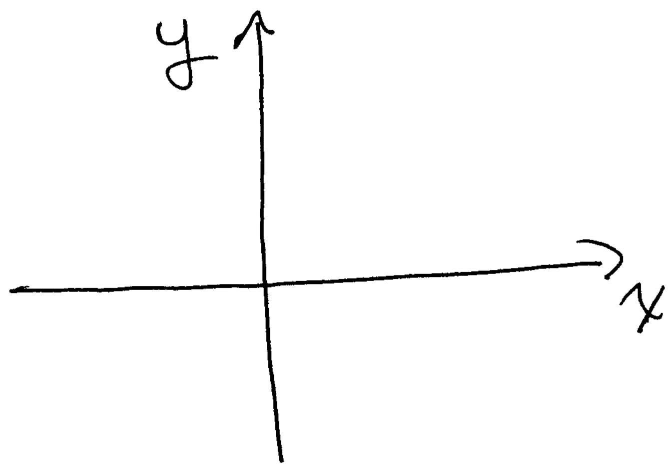
<u>Note:</u> A <u>saddle point</u> is a critical point. However, f has neither a max nor a min at the point.

Calculus I

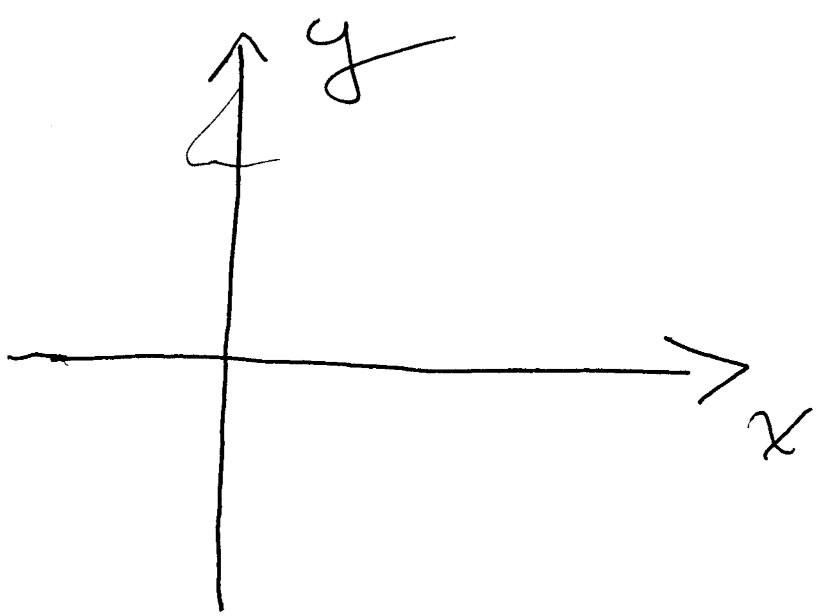
$y = x^2$



$y = -x^2$



$y = x^3$



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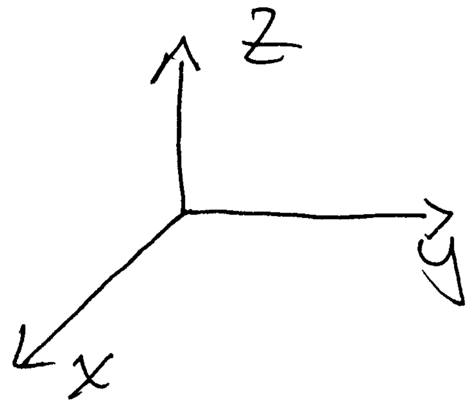
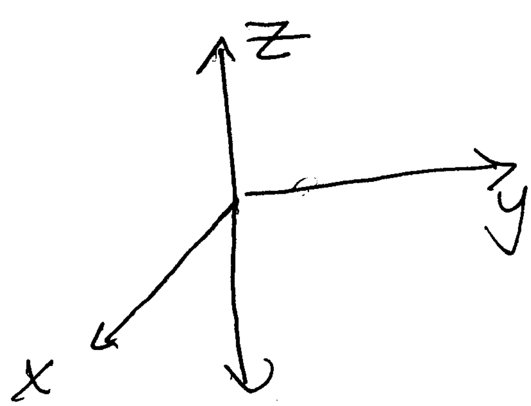
$$z = x^2 + y^2$$



$$z = -(x^2 + y^2)$$



$$z = y^2 - x^2$$

Maple example

12-7

Concavity from Calculus I

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What does f_{xx} represent?

what does f_{yy} represent?

what does $f_{xx}(a,b)f_{yy}(a,b)$ tell us at a critical point?

<u>Note</u>: Simply looking at the sign of $f_{xx}(a,b) f_{yy}(a,b)$ is an insufficient test for extrema.

$$f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

Is called the Discriminant of f at (a, b).

From Theorem 11 (see textbook for complete statement) 9

At (a,b), $f_x(a,b) = f_y(a,b) = 0$

1) f has a local maximum at (a,b) if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a,b)

2) f has a local minimum at (a,b) if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$ at (a,b)

3) f has a saddle point at (a,b) if $f_{xx}f_{yy} - f_{xy}^2 < 0$ at (a,b)

4) If $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (a,b), the test is inconclusive.

example:
consider the function

$$f(x,y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$$

a) show that $(\frac{2}{3}, \frac{4}{3})$ is the only critical point.

b) verify that the discriminant function is $f_{xx} f_{yy} - f_{xy} = (-10)(-4) - 2^2$
$= 36$

c) use the sign of f_{xx} at $(\frac{2}{3}, \frac{4}{3})$ and the discriminant function to classify the critical point $(\frac{2}{3}, \frac{4}{3})$.