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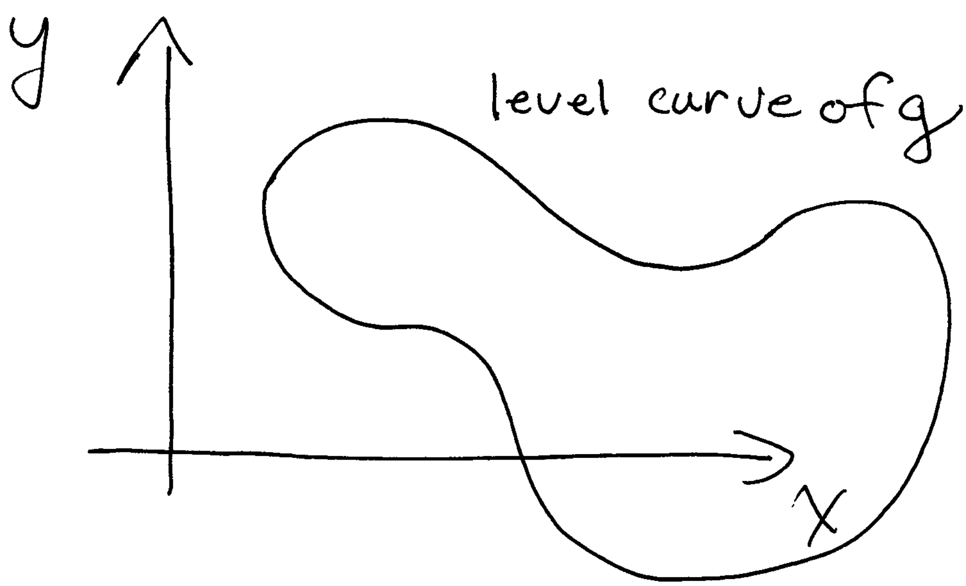

Suppose we want to optimize $f(x,y)$ restricted to the level curve above.

Let $\vec{r}(t)$ be a parameterization of this level curve

$$(*) \quad D_{\frac{\vec{r}'}{|\vec{r}'|}} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \frac{r'(t)}{|\vec{r}'(t)|}$$

How can we use this to test whether a point on the level curve is where a max or min for f occurs?

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<u>Conclusion:</u> Find points on level curve where

$$D_{\frac{\vec{r}'}{|\vec{r}'|}} f(\vec{r}(t)) = 0$$

— or —

$$\vec{\nabla} f \cdot \vec{r}\,' = 0$$

— or —

since $\vec{\nabla} g \perp r'$,

$\vec{\nabla} f = \lambda \vec{\nabla} g$ (parallel)

Lagrange multiplier method

Solve the system

$$\begin{cases} \vec{\nabla f} = \lambda \vec{\nabla g} \\ g(x,y) = c \end{cases}$$

for (x, y, λ). We often discard λ.

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ex Find the points on the ellipse $x^2+2y^2=1$ where $f(x,y)=xy$ has its extreme values

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$$\begin{cases} \nabla f = \lambda \nabla g \\ x^2+2y^2=1 \end{cases}$$

Find the minimum distance from the surface to the origin.

$$\begin{cases} f(x,y,z) = x^2+y^2+z^2 \\ g(x,y,z) = x^2+y^2-z^2 = 1 \end{cases}$$