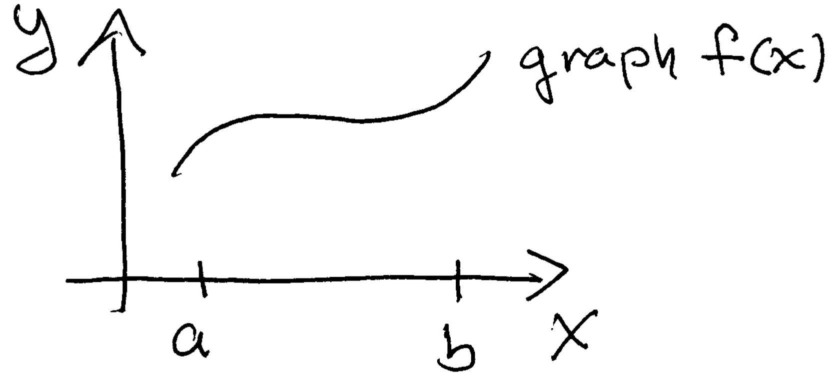
Section 13.1

Calculus II



$$\int_a^b f(x)\,dx \overset{\text{def}}{=} \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

If $f \geq 0$ (at least on $[a,b]$)
then the integral gives the area under the graph and above the x-axis

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Integration of $f(x,y)$ over rectangles

For functions of two variables, we again partition the domain

$$R = [a,b] \times [c,d]$$

$$\sum_{k=1}^{n} f(x_k, y_k)\,\Delta A_k \qquad \iint_R f(x,y)\,dA$$

If $f(x,y) \geq 0$ for
$a \leq x \leq b$ and $c \leq y \leq d$,
then $\iint_R f(x,y)\,dA$ is the
volume between the surface
$z = f(x,y)$ and region R.

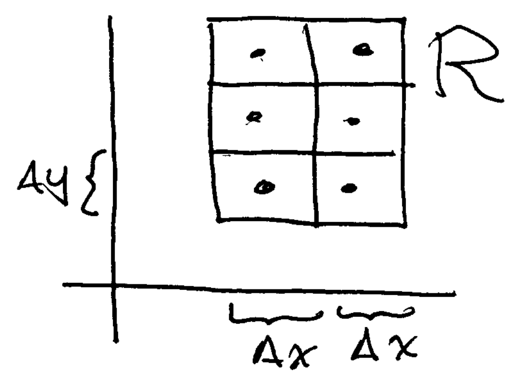
Fubini's Theorem

$$\iint_R f(x,y)\,dA = \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} f(x_k, y_k)\,\Delta A_k}_{\text{reorganize}}$$

example:

f

Reorganize sums



$$\sum_{k=1}^{6} f(x_k, y_k)\,\Delta A_k =$$

$$= f(\tilde{x}_1, \tilde{y}_1)\Delta x\Delta y + f(\tilde{x}_1, \tilde{y}_2)\Delta x\Delta y + f(\tilde{x}_1, \tilde{y}_3)\Delta x\Delta y$$
$$+ f(\tilde{x}_2, \tilde{y}_1)\Delta x\Delta y + f(\tilde{x}_2, \tilde{y}_2)\Delta x\Delta y + f(\tilde{x}_2, \tilde{y}_3)\Delta x\Delta y$$

$$= f(\tilde{x}_1, \tilde{y}_1)\Delta x\Delta y + f(\tilde{x}_2, \tilde{y}_1)\Delta x\Delta y$$
$$+ f(\tilde{x}_1, \tilde{y}_2)\Delta x\Delta y + f(\tilde{x}_2, \tilde{y}_2)\Delta x\Delta y$$
$$+ f(\tilde{x}_1, \tilde{y}_3)\Delta x\Delta y + f(\tilde{x}_2, \tilde{y}_3)\Delta x\Delta y$$

$$\iint_R f(x,y)\,dA$$

$$= \lim_{n\to\infty} \sum_{k=1}^{\infty} f(x_k, y_k)\,\Delta A_k$$

$$= \lim_{l\to\infty} \lim_{m\to\infty} \sum_{i=1}^{l} \sum_{j=1}^{m} f(\tilde{x}_i, \tilde{y}_j)\,\Delta x\,\Delta y$$

$$= \lim_{l\to\infty} \sum_{i=1}^{l} \left(\lim \sum_{j=1}^{m} f(\tilde{x}_i, \tilde{y}_j)\,\Delta y\right)\Delta x$$

$$= \lim_{l\to\infty} \sum_{i=1}^{l} \int_c^d f(\tilde{x}_i, y)\,dy$$

$$= \int_a^b \left(\int_c^d f(x,y)\,dy\right)dx$$

— similarly —

$$\iint_R f(x,y)\,dA$$

$$= \int_c^d \left(\int_a^b f(x,y)\,dx\right)dy$$

Iterated Integrals

Fubini's Theorem (See textbook for more hypotheses)

If $f(x,y)$ is continuous on $R=[a,b]\times[c,d]$, then

$$\iint_R f(x,y)\,dA = \int_c^d\int_a^b f(x,y)\,dx\,dy$$
$$= \int_a^b\int_c^d f(x,y)\,dy\,dx$$

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ex: Evaluate $\iint_R e^{x-y}\,dA$

on $R: \begin{aligned} 0 \le x \le \ln 2 \\ 0 \le y \le \ln 2 \end{aligned}$

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ex: Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and bounded below by $R: -1 \le x \le 1$
$-1 \le y \le 1$