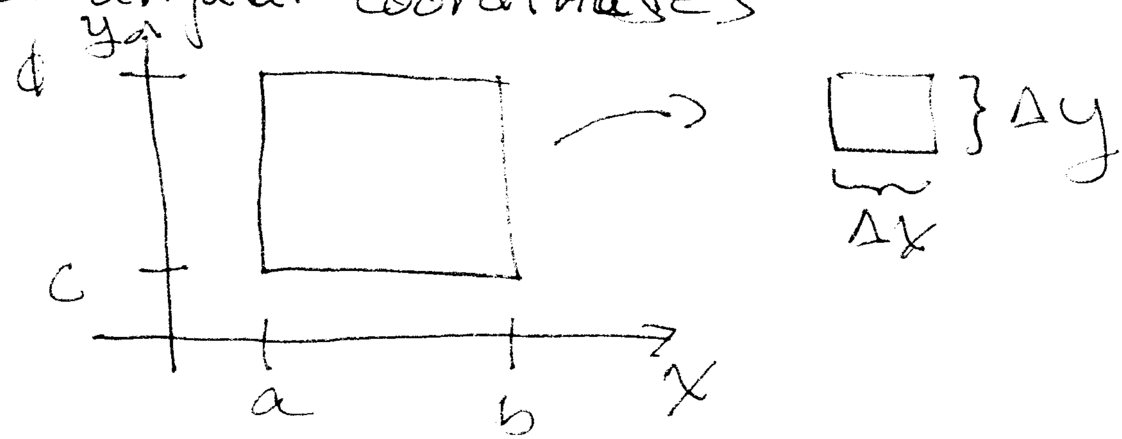
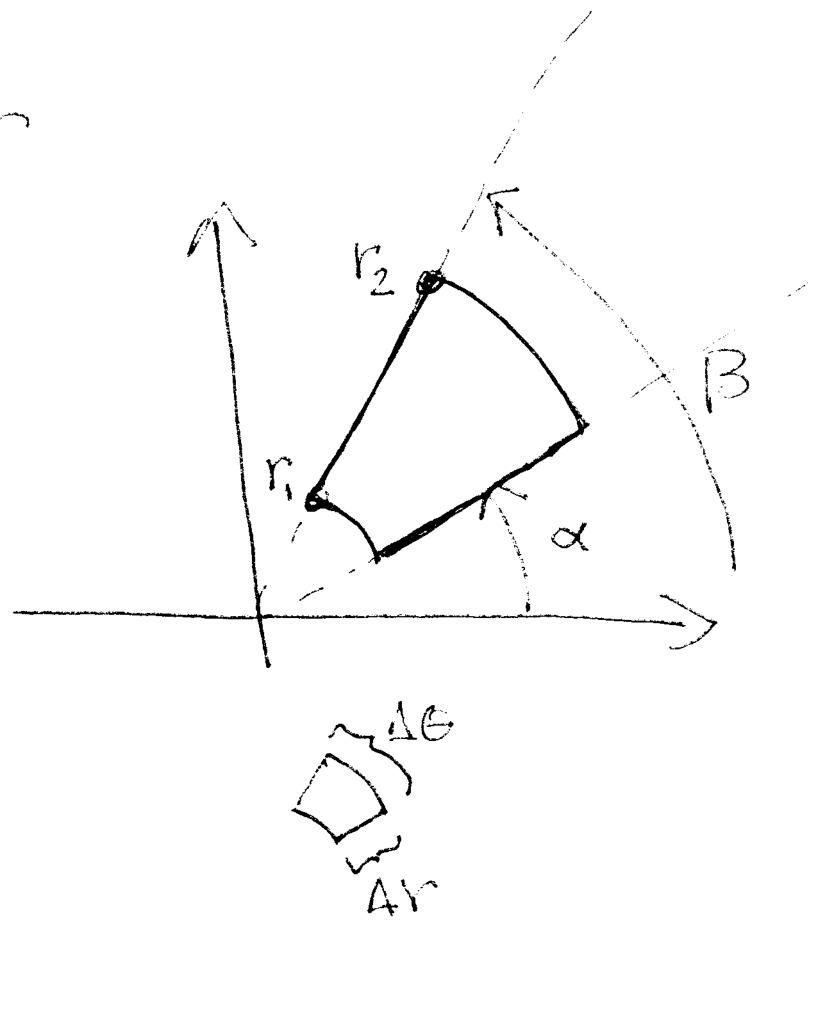
13.4 Double Integrals in Polar Form

Fundamental region in rectangular coordinates



Polar



13.4

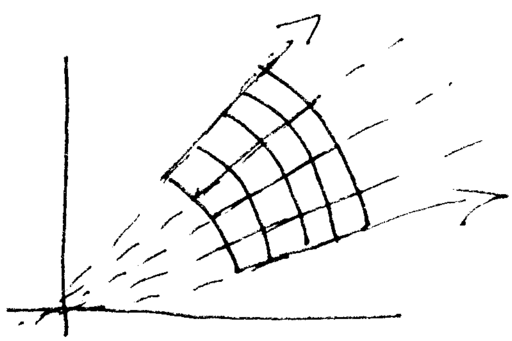
$$\text{Area} = \frac{1}{2} r_2^2 \Delta\theta - \frac{1}{2} r_1^2 \Delta\theta$$

replace r_2 with $r_2 = r_1 + \Delta r$

$$\text{Area} = \frac{1}{2} (r_1 + \Delta r)^2 \Delta\theta - \frac{1}{2} r_1^2 \Delta\theta$$

= [ ]

= [ ]



Assume
m divisions of r
n divisions of θ

$$\Delta r = \frac{r_2 - r_1}{m} \quad \text{and} \quad \Delta\theta = \frac{\beta - \alpha}{n}$$

so $m =$ [ ] $n =$ [ ]

$$mn = \frac{\text{stuff}}{[\ \ ]}$$

13.4

Fundamental Area

$$\boxed{r_i \Delta r \Delta \theta}$$

$$\sum_{j=1}^{n} \sum_{i=1}^{m} f(r_i \cos\theta_j, r_i \sin\theta_j)\, r_i \Delta r \Delta \theta$$

$$\xrightarrow{n, m \to \infty} \iint_R f(\underbrace{r\cos\theta}_{x}, \underbrace{r\sin\theta}_{y})\, r\,dr\,d\theta$$

Textbook $\iint f(r, \theta)\, r\,dr\,d\theta$