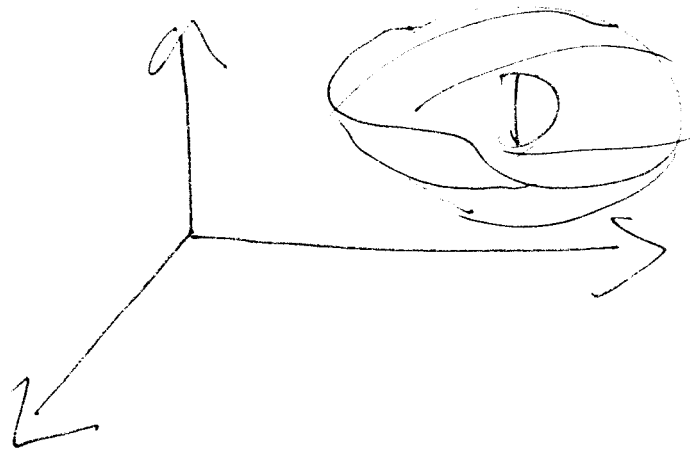


## 13.5 Triple Integrals



Integration over a solid  
D in space

$$S_n \stackrel{\text{def}}{=} \sum_{k=1}^n F(x_k, y_k, z_k) \Delta V_k$$

$$\lim_{n \rightarrow \infty} S_n = \iiint_D F(x, y, z) dV$$

(If the limit exists)

### 13.5

As with double integrals, triple integrals can often be computed using iterated integrals. Three, namely.

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For example,

$$\iiint_D F(x, y, z) dV = \int \int_{g(x)}^{g(x)} \int_{z=f_1(x, y)}^{z=f_2(x, y)} F(x, y, z) dz dy dx$$

B.5

(14)

Ex = Find the volume of the solid  $V$  bounded by

$D$

$$z = 8 - x^2 - yz$$

and

$$z = x^2 + y^2$$

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$$\iiint_D 1 \, dV$$

ex: sketch the region of integration

$$\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz dy dx$$

Sketch the region of integration

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz dx dy$$

