13.7 cylindrical coordinates

$(x, y, z) \longleftrightarrow (r, \theta, z)$

where as in polar coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\theta = \arctan\left(\frac{y}{x}\right)$

ex:

Describe the region of integration

(6) $$\int_0^{2\pi}\int_0^{1}\int_{-1/2}^{1/2}(r^2\sin^2\theta+z^2)\,dz\,r\,dr\,d\theta$$

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useful formulas

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Ex: Describe the region of integration

(9) $$\int_0^1 \int_0^{\sqrt{z}} \int_0^{2\pi} (r^2\cos^2\theta + z^2) r \, d\theta \, dr \, dz$$

Useful formulas

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

(4)

Consider

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\, dz\, dy\, dx$$

Convert to cylindrical and evaluate.

13.7

ex: A solid E lies within the cylinder $x^2+y^2=1$, below the plane $z=4$ and above the paraboloid $z=1-x^2-y^2$.

The density at any point is proportional to its distance from the axis of the cylinder.

Find the mass of E.

ex:

Density $\delta(x,y,z) = \lambda\sqrt{x^2+y^2}$

$$\text{mass} = \iiint_E \lambda\sqrt{x^2+y^2}\, dV$$

$$= \int_0^{2\pi}\int_0^1\int_{1-r^2}^{4} (\lambda r)\, r\, dz\, dr\, d\theta$$