Math 430/630

Partial Solutions to practice Questions.

(2) $L(\vec{0}) = L(\vec{0}+\vec{0}) \overset{\text{use Linearity}}{=} L(\vec{0}) + L(\vec{0})$

$\cancel{L(\vec{0})} = \cancel{L(\vec{0})} + L(\vec{0})$

$\vec{0} = L(\vec{0})$ is required

(3) $\left(\begin{array}{ccc|c} 1 & -1 & 0 & b_1 \\ 2 & 3 & 0 & b_2 \\ 3 & 2 & 2 & b_3 \\ 6 & 4 & 2 & b_4 \end{array}\right) \overset{\text{row}}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & 0 & b_1 \\ 0 & 5 & 0 & b_2 - 2b_1 \\ 0 & 0 & 2 & b_3 - b_1 - b_2 \\ 0 & 0 & 0 & b_4 - b_1 - b_2 - b_3 \end{array}\right)$

$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ is in the span if and only if $b_4 - b_1 - b_2 - b_3 = 0$ //

(5) $c_1 v_1 + c_2 v_2 + c_3 w + c_4 v_3 = \vec{0}$

$c_1 v_1 + c_2 v_2 + c_3(3v_1 + v_2) + c_4 v_3 = \vec{0}$

$(c_1 + 3c_3) v_1 + (c_2 + c_3) v_2 + c_4 v_3 = \vec{0}$

Any nontrivial sol'n of

$\begin{cases} c_1 + 3c_3 = 0 \\ c_2 + c_3 = 0 \\ c_4 = 0 \end{cases}$ shows the set $\{v_1, v_2, w, v_3\}$ is linearly dependent.

(6) Decide whether the following are subspaces. Make this determination by checking the two closure properties.

- The subset $R(A)$ of $\mathbb{R}^3$ where A is a 3×3 matrix.

  Start on Solution: Let $\vec{y_1}, \vec{y_2} \in R(A)$ and let r be a real number. By hypothesis there exist $\vec{x_1}, \vec{x_2} \in \mathbb{R}^3$ with the property that $A\vec{x_1} = \vec{y_1}$ and $A\vec{x_2} = \vec{y_2}$. To complete the argument you must convince me that both $\vec{y_1} + \vec{y_2}$ and $r\vec{y_1}$ are in $R(A)$.

- The subset $N(A)$ of $\mathbb{R}^3$ where A is a 3×3 matrix.

consider $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

- All vectors $[b_1 \quad b_2 \quad b_3]^T$ in $\mathbb{R}^3$ with $b_1 b_2 b_3 = 0$
- All vectors $[b_1 \quad b_2 \quad b_3]^T$ in $\mathbb{R}^3$ with $b_1 = 0$ — yes
- All vectors $[x_1 \quad x_2 \quad x_3]^T$ in $\mathbb{R}^3$ with $x_1 + x_2 + x_3 = 0$

  Hint: You can do this directly but here is a fast way: consider $A\vec{x} = 0$ where $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ then think about $N(A)$.

- All vectors $[x_1 \quad x_2 \quad x_3]^T$ in $\mathbb{R}^3$ with $x_1 + x_2 + x_3 = 3$

  Hint: consider $A\vec{x} = 3$ where $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$\left.\begin{matrix} Ax = 3 \\ Ay = 3 \end{matrix}\right\}$ but $A(x+y) = 6$ No.

consider $(-1)\begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$

- All vectors $[b_1 \quad b_2 \quad b_3]^T$ in $\mathbb{R}^3$ with $b_1 \le b_2 \le b_3 = 0$

(7) Consider the basis $S = \{t+2, t+1, t^2+t\}$ for the vector space of polynomials of degree less than or equal to 2.
 (a) Find $[at^2 + bt + c]_S$.
 (b) Let T be given by $T(p(t)) = p(t) + 2\frac{dp}{dt}(t)$. Find the coordinate matrix $[T]_S$ for T.

like HW 4.4.5

(8) Consider a nonzero vector $v \in \mathbb{R}^3$. Explain how you would extend the set $\{v\}$ in a systematic way to a basis for $\mathbb{R}^3$.

(9) Section 4.7: Exercises 1, 2, 7, 17

(10) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

by elementary row operations $[\,A \mid I\,] \sim \cdots \sim [\,I \mid A^{-1}\,]$.

$$A^{-1} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

⑦

(a) $$[at^2+bt+c]_{S'} = \begin{pmatrix} a-b+c \\ -2a+2b-c \\ a \end{pmatrix}$$

(b) $$[T]_{S'} = \left([T(t+2)]_{S'} \,\middle|\, [T(t+1)]_{S'} \,\middle|\, [T(t^2+t)]_{S'} \right)$$

$$= \left([t+4]_{S} \,\middle|\, [t+3]_{S'} \,\middle|\, [t^2+5t+2]_{S'} \right)$$

$=$ ... use part (a)