MATH 430/630 Exam I Study Questions

- Section 4.1: Determine whether a given subset of a vector space is a subspace
- Section 4.2: Determine four fundamental subspaces of a given matrix A
- Section 4.3: Linear Independence and Bases
- Section 4.4: Dimensions of four fundamental subspaces; Rank Plus Nullity Theorem
- Section 4.7: Determine whether a given map between vector spaces is a Linear Transformation
- Section 4.7: Coordinate Vectors; Coordinate Matrices

(1) Find the four fundamental subspaces by exhibiting a basis for each subspace.

$$A = \begin{bmatrix} 1 & 0 & -1 & 3 & -1 \\ 1 & 0 & 0 & 2 & -1 \\ 2 & 0 & -1 & 5 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

(2) Suppose V is a vector space and consider a function $L : V \to V$. If $L(\vec{0}) \neq \vec{0}$, can L be a linear transformation?

(3) Let $T = \{\vec{v_1}, \vec{v_2}, \vec{v_3}\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix} \right\}$

- Find—if possible—c_1, c_2, c_3 such that $[1 \quad 1 \quad 6 \quad 10]^T = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3}$
- Find—if possible—c_1, c_2, c_3 such that $[1 \quad 1 \quad 1 \quad 5]^T = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3}$

(4) Determine whether the set of vectors $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \\ 2 \end{bmatrix} \right\}$ is linearly independent or dependent.

(5) Suppose $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ is a linearly independent set of vectors. Determine whether the set of four vectors

$$\{\vec{v_1}, \vec{v_2}, \overbrace{3\vec{v_1} + \vec{v_2}}^{\vec{w}}, \vec{v_3}\}$$

is also a linearly independent set. Do not simply say yes or no. Give an explanation. Prove to me that you know the definitions of *linear independence/dependence*. A well chosen equation involving some constants c_1, c_2, c_3, c_4 and the vectors would $\{\vec{v_1}, \vec{v_2}, \vec{w}, \vec{v_3}\}$ increase the likelyhood that I'll believe you.

(6) Decide whether the following are subspaces. Make this determination by checking the two closure properties.

- The subset $R(A)$ of $\mathbb{R}^3$ where A is a 3×3 matrix.

  Start on Solution: Let $\vec{y_1}, \vec{y_2} \in R(A)$ and let r be a real number. By hypothesis there exist $\vec{x_1}, \vec{x_2} \in \mathbb{R}^3$ with the property that $A\vec{x_1} = \vec{y_1}$ and $A\vec{x_2} = \vec{y_2}$. To complete the argument you must convince me that both $\vec{y_1} + \vec{y_2}$ and $r\vec{y_1}$ are in $R(A)$.

- The subset $N(A)$ of $\mathbb{R}^3$ where A is a 3×3 matrix.
- All vectors $[b_1 \quad b_2 \quad b_3]^T$ in $\mathbb{R}^3$ with $b_1 b_2 b_3 = 0$
- All vectors $[b_1 \quad b_2 \quad b_3]^T$ in $\mathbb{R}^3$ with $b_1 = 0$
- All vectors $[x_1 \quad x_2 \quad x_3]^T$ in $\mathbb{R}^3$ with $x_1 + x_2 + x_3 = 0$

  Hint: You can do this directly but here is a fast way: consider $A\vec{x} = 0$ where $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ then think about $N(A)$.

- All vectors $[x_1 \quad x_2 \quad x_3]^T$ in $\mathbb{R}^3$ with $x_1 + x_2 + x_3 = 3$

  Hint: consider $A\vec{x} = 3$ where $A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

- All vectors $[b_1 \quad b_2 \quad b_3]^T$ in $\mathbb{R}^3$ with $b_1 \le b_2 \le b_3 = 0$

(7) Consider the basis $S = \{t+2, t+1, t^2+t\}$ for the vector space of polynomials of degree less than or equal to 2.
   (a) Find $[at^2 + bt + c]_S$.
   (b) Let T be given by $T(p(t)) = p(t) + 2\frac{dp}{dt}(t)$. Find the coordinate matrix $[T]_S$ for T.

(8) Consider a nonzero vector $v \in \mathbb{R}^3$. Explain how you would extend the set $\{v\}$ in a systematic way to a basis for $\mathbb{R}^3$.

(9) Section 4.7: Exercises 1, 2, 7, 17

(10) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

by elementary row operations $[\,A \,|\, I\,] \sim \cdots \sim [\,I \,|\, A^{-1}\,]$.