Cylinders are surfaces generated by moving a straight line along a curve while keeping the direction of the line constant.

Most of the cylinders we will encounter in this class will be generated by a line parallel to the $x$-axis or $y$-axis or the $z$-axis.

\[
x^2 + y^2 = 16
\]
\[
x + z^2 = 16
\]
\[
y - x^2 = 2
\]
The basic surfaces we consider are ellipsoids, paraboloids, elliptic cones, hyperboloids.
Ellipsoid

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]
Elliptic Paraboloid

\[ \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]
Hyperbolic paraboloid (saddle)

\[ \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \]
Hyperboloid of one sheet
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \]
cone  \[ \frac{z^2}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \]
Hyperboloid of two sheets

\[-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\]
<table>
<thead>
<tr>
<th>Surface</th>
<th>Equation</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 )</td>
<td>Cone</td>
<td>( \frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2} )</td>
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<tr>
<td></td>
<td>All traces are ellipses. If ( a = b = c ), the ellipsoid is a sphere.</td>
<td></td>
<td>Vertical traces are ellipses. Vertical traces in the planes ( x = k ) and ( y = k ) are hyperbolas if ( k \neq 0 ) but are pairs of lines if ( k = 0 ).</td>
</tr>
<tr>
<td>Elliptic Paraboloid</td>
<td>( \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} )</td>
<td>Hyperboloid of One Sheet</td>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 )</td>
</tr>
<tr>
<td></td>
<td>Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.</td>
<td></td>
<td>Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.</td>
</tr>
<tr>
<td>Hyperbolic Paraboloid</td>
<td>( \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} )</td>
<td>Hyperboloid of Two Sheets</td>
<td>( -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 )</td>
</tr>
<tr>
<td></td>
<td>Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where ( c &lt; 0 ) is illustrated.</td>
<td></td>
<td>Horizontal traces in ( z = k ) are ellipses if ( k &gt; c ) or ( k &lt; -c ). Vertical traces are hyperbolas. The two minus signs indicate two sheets.</td>
</tr>
</tbody>
</table>