Estimate the volume of the solid that lies below the surface \( z = xy \) and above the following rectangle.

\[ R = \{(x, y) \mid 10 \leq x \leq 16, \ 2 \leq y \leq 6\} \]

(a) Use a Riemann sum with \( m = 3, \ n = 2 \), and take the sample point to be the upper right corner of each square.

(b) Use the Midpoint Rule to estimate the volume of the solid.
A 20-ft-by-30-ft swimming pool is filled with water. The depth is measured at 5-ft intervals, starting at one corner of the pool, and the values are recorded in the table. Estimate the volume of water using the Midpoint Rule with \( m = 2 \) and \( n = 3 \).

<table>
<thead>
<tr>
<th>x,y</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>8</td>
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<td>5</td>
<td>5</td>
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<td>2</td>
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<td>5</td>
<td>8</td>
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<tr>
<td>20</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
Evaluate the double integral by first identifying it as the volume of a solid.

\[ \int_{R} \int (5 - x) \, dA, \quad R = \{(x, y) \mid 0 \leq x \leq 5, \, 0 \leq y \leq 4\} \]
\[ z = 5 - x \geq 0 \text{ for } 0 \leq x \leq 5, \text{ so we can interpret the integral as the volume of the solid } S \text{ that lies below the plane } z = 5 - x \text{ and above the rectangle } [0, 5] \times [0, 4]. \] 

S is a triangular cylinder whose volume is 
\[ 4 \cdot \text{(area of triangle)} = 4 \times \left( \frac{1}{2} \cdot 5 \cdot 5 \right) = 50. \] 

Thus \( \iiint_S (5 - x) \, dA = 50 \)

Assume \( a = 5 \)
\( b = 4 \)
\( c = 5 \)
Exercise

Estimate the volume of the solid that lies below the surface \( z = xy \) and above the rectangle

\[ R = \{(x, y) \mid 10 \leq x \leq 16, 2 \leq y \leq 6\} \]

a) Use a Riemann sum with

- \( m = 3 \) (divisions of \( 10 \leq x \leq 16 \))
- \( n = 2 \) (divisions of \( 2 \leq y \leq 6 \))

Take sample points from upper right corners

Answer: 1680
Estimate the volume of the solid that lies below the surface \( z = xy \) and above the rectangle

\[ R = \{(x,y) \mid 10 \leq x \leq 16, 2 \leq y \leq 6\} \]

b) use the midpoint rule to estimate the volume. Again, \( m = 3, n = 2 \).

Answer: 1248
A 20 ft by 30 ft swimming pool is filled with water. The depth is measured at 5 ft intervals starting at one corner of the pool, and the values are recorded in the table. Estimate the volume of water using the midpoint rule.

\[ m = 2 \text{ (x divisions) } \quad n = 3 \text{ (y divisions) } \]

\[
V = \sum_{i=1}^{2} \sum_{j=1}^{3} f(x_i, y_j) \Delta A
\]

\[
= \Delta A \left[ f(x_1, y_1) + f(x_1, y_2) + f(x_1, y_3) \\
+ f(x_2, y_1) + f(x_2, y_2) + f(x_2, y_3) \right]
\]

\[
= (100) \left[ f(5, 5) + f(5, 15) + f(5, 25) \\
+ f(15, 5) + f(15, 15) + f(15, 25) \right]
\]

\[
= (100) [4 + 5 + 9 + 3 + 5 + 7]
\]

\[
= 3300 \text{ volume}
\]
plot(30, x=0..20, view=0..30, gridlines=true, font=[Times, bold, 16], labelfont=[Times, bold, 18]);
4) Evaluate the double integral by first identifying it as the volume of a solid.

\[ \iint_{R} (5-x) \, dA \]

\[ R = \{(x,y) \mid 0 \leq x \leq 5, \ 0 \leq y \leq 4\} \]
> restart;

> plot3d(5-x, x=0..5, y=0..5, axes=boxed, font=[Times, bold, 16], labelfont=[Times, bold, 18]);