Answers

—Sample Questions for Final Exam from Sections 16.4, 16.8 and 16.9—

(1) Let \( \vec{F} = xy^2 \hat{i} + xy \hat{j} + 3 \hat{k} \) and consider the surface \( S : x^2 + y^2 + z^2 = 49 \). If \( D \) is the ball enclosed by \( S \), then which of the triple integrals below is equal to the surface integral \( \iint_S \vec{F} \cdot \vec{n} \, dS \)? Use the Divergence Theorem.

(a) \( \iiint_D (x + y^2 + xz) \, dV \)  
(b) \( \iiint_D (yz) \, dV \)  
(c) \( \iiint_D (y^2 + x) \, dV \)  
(d) \( \iiint_D (1 + x + 2y) \, dV \)  
(e) none of these

(1) _____

\[
\iiint_S \vec{F} \cdot \vec{n} \, dS = \iiint_D \left( \frac{\partial}{\partial x} (xy^2) + \frac{\partial}{\partial y} (xy) + \frac{\partial}{\partial z} (3) \right) \, dV
\]
(2) Let \( \vec{F} = P\vec{i} + Q\vec{j} = (3 + 2x)\vec{i} + (7x + y)\vec{j} \).

Consider the line integral \( \oint_C \vec{F} \cdot \vec{T} \, ds \) over a counter-clockwise oriented simple closed curve \( C \) which bounds a region \( R \) in the \( xy \)-plane.

If the area of the region \( R \) in the \( xy \)-plane is 3, then by Green’s Theorem the value of \( \oint_C \vec{F} \cdot \vec{T} \, ds \) is

(a) 9  (b) 3  (c) 7  (d) 21  (e) none of these