restart;
1st constraint equation $x^2+y^2+z^2=3$; Below we consider the upper hemisphere.
We will plot a normal vector and tangent plane to this level surface at $P(1,1,1)$;
2nd constraint equation $x=1$;
We will plot a normal vector to this second surface at $P(1,1,1)$;

> g1:=sqrt(3-x^2-y^2);

\[ g_I := \sqrt{3 - x^2 - y^2} \]  

(1)

> P := [1, 1, 1]:

> with(Student[VectorCalculus]):

> N := evalVF(Gradient(z-g1), `<,>`(P));

\[ N := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \]  

(2)

Z:=3-x-y;#This is the equation of the tangent plane.

\[ Z := 3 - x - y \]  

(3)

> with(plots):

> p[1]:=plot3d(g1, x = -sqrt(3)..sqrt(3), y = -sqrt(3-x^2) .. sqrt(3-x^2), color = red,axes=boxed,grid=[80,80]):


> p[3]:=plot3d(Z, x = 0..2, y = -2 .. 2,color = blue):

> display3d(p[1], p[2], p[3], axes = boxed, scaling = constrained, view = 0 .. 2, labels = [x, y, z], orientation = [25, 80, 0]);
> VectorField( <1, 0, 0>, 'cartesian'[x,y,z] );

\( \vec{e}_x \)

> N2 := evalVF(VectorField( <1, 0, 0>, 'cartesian'[x,y,z] ), `\langle`, `(P)`); # normal vector rooted at 
P

\[
N2 := \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

> p[4]:=plot3d([1,y,z], y = -2..2, z = 0..2, color = yellow):

> display3d(p[4], p[5], axes = boxed, scaling = constrained, view = -1 .. 2, labels = [x, y, z], orientation = [25, 80, 0]);
IntersectionCurve := spacecurve([1, sqrt(2)*cos(t), sqrt(2)*sin(t)], t=0..Pi, color=black, thickness=5):
display3d(p[1], p[2], p[3], p[4], p[5], IntersectionCurve, axes = boxed, scaling = constrained, view = -1 .. 2, labels = [x, y, z], orientation = [25, 80, 0]);

grad f = (1, 2, 3); grad g1(1,1,1) = (2, 2, 2) and grad g2 = (1, 0, 0) and importantly grad g1(1,1,1) and grad g2 = (1, 0, 0) are linearly independent.
grad f is not a linear combination of (2, 2, 2) and (1, 0, 0). Therefore grad f has a component in the nonempty intersection of the tangent planes of the two constraint surfaces. Hence, there should be a path in the intersection of the two constraint surfaces on which value of f increase above the value of f at P. Thus P is not the location of a maximum. Similarly, P is not the location of a minimum.