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End
Assumptions:

If \( n = 1 \) then \( C = 5 \).

If \( n > 1 \), then
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C = \begin{cases} 
4 + n & \text{if the batch passes} \\
4 + 5n & \text{if the batch doesn't pass} 
\end{cases}
\]

Finally, \( A = \frac{\text{average}(C)}{n} = \frac{E(C)}{n} \).

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Solve the model. We want to minimize $A = E(C)/n$. 

The expected value of $C$ is $E(C) = p(4 + n) + (1 - p)(4 + n + \lceil n \rceil)$ where $p$ is the probability that all $n$ diodes in the batch are good.

This simplifies to $E(C) = 4 + 6n - 5p$ and so $A(n) = E(C)n = 4n + 6 - 5p$. 

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In particular, we optimize \( A \) with respect to \( n \) and then afterwards consider the sensitivity of our results to the assumed failure rate \( q = 0.003 \).
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