Definition: An equation containing the derivatives of one or more dependent variables with respect to one or more independent variables is said to be a differential equation.

Examples:
we will classify DE's by order and linearity.

order

Linearity (by form)

\[ a_n(x) \frac{dy}{dx} + a_0(x) y = g(x) \]

\[ a_2(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x) y = g(x) \]

examples of linear and non-linear...

why do we classify?
Solutions of a DE.

A solution is a function (and an interval of definition) which "satisfies" the DE.

Examples

a) Verify that \( y = \sin 3x \) is a solution of

\[
\frac{d^2y}{dx^2} + 9y = 0
\]

but that \( y = e^{3x} \) is not.
b) Verify that $y = \frac{1}{x}$ is a solution of $\frac{dy}{dx} + \frac{1}{x} y = 0$.

What is the interval of definition?
c) Find \( r \) such that \( e^{rx} \)
is a sol'n of \( y'' - 5y' + 6y = 0 \).
explicit versus implicit solutions.

\[ xy + y^2 = 0 \] is an implicit solution of the non-linear equation

\[ (x+2y)y' + y = 0 \]