

# Isomorphisms

In the library you will find a Mystery Group. Your job is to determine which group in the library it is isomorphic to and to construct an isomorphism to that group. Here are some guiding questions:<sup>1</sup>

- How many elements of order 2 does the mystery group have?
- Which groups have the same number of order 2 elements?
- Which group  $\mathcal{G}$  in the library must be isomorphic to the Mystery Group?
- Define your isomorphism to map the center of the Mystery group to the center of  $\mathcal{G}$ .
- At this point in order to complete your definition for the isomorphism you must look for some common structure among the groups. For example, isomorphisms must preserve orders of elements. Try mapping some elements of order 6 to other elements of order 6 to get an isomorphism. If this is not enough to make things work, then you will need to be a bit more sophisticated with these ideas - such as mapping non-cyclic subgroups to isomorphic subgroups in the two groups.

**VAGUE OPEN PROBLEM: IS there a meaningful way to assign words to the elements of a group?**

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<sup>1</sup>Examples like this are easy to construct. In the main menu you can use the replace commands to change around the names of the elements. You can also use your mouse to move columns and reorder the elements in the table. (The rows then also get moved automatically) Be sure to use “save as” to save your group as a new file. Another way to create Mystery groups of smaller size is to realize them as subgroups or quotient groups of larger groups which can then be exported.