**WHAT DID WE DISCOVER? (THE HIGHLIGHTS!)**

**STUDENTS - PUPPIES AND KITTENS**

We noticed the following basic facts:

- If it is your turn with only one pile left then you can win.
- If it is your turn with two pile that have an equal number you can win.
- If you leave your opponent two in one pile and one in the other then you can win.

So we wondered what is the set of ordered pairs \( W = \{(m, n) \Rightarrow m \text{ Kittens} \& n \text{ Puppies}\} \) defined by the rule that if it is your opponents turn with an ordered pair in \( W \) then you can win. Therefore, \((2, 1), (1, 2) \in W\). A winning strategy would consist of getting the result to an element of \( W \) when it is your turn. \( W \) must have the property that no move can take you from an element of \( W \) to another element of \( W \) but any move from an element in \( W \) will take you to a position by which you can move to \( W \) when it is your turn again. We started making a list of these elements of \( W \) and putting them on a grid as shown below. Is this correct? Notice that legal moves always cause movement toward the origin either horizontally, vertically, or on a line of slope 1. Is there a formula to describe all the safe positions on the grid?
STUDENTS Abbot and Costello Numbers

The tricks involve changing the algorithms ever so slightly so that they have consistency among addition, division and multiplication but are staggeringly incorrect. They “show” that 7 goes into 28, 13 times by doing 7 into 8 once (it won’t fit into 2) and then with one left over we have 7 into 21 3 times. So the partial results of remainder 1 and 3 times gives us 13. Thus to build another problem we need for **DIVISION**

- A single digit divisor \( n \).
- A two digit dividend \( ab \).
- A two digit quotient \( cd \).
- \( a < n \)
- \( b \geq n \)
- The number with first digit \( a \) and second digit \( n - b \) must be divisible by \( n \) with quotient \( q \)
- The result is \( 1q \).

The multiplication trick then proceeds by performing \((n \times q) + (n \times 1)\) (instead of \( n \times 10 \)). This is \( n(q + 1) = 10a + b \) and it is equivalent to the above formulation. Finally, doing multiplication as repeated addition works the same way. Therefore all three calculations rely on the same trick. You can for example “show” that 4 times 18 is 36. Questions remain - what tricks can be used on three digit numbers? This shows a quotient which is too big or a product which is too small. Does it ever work out the other way where the quotient is too small and the product too big? What other tricks are possible?
TEACHER’s CELL PHONES

The every $n$th floor strategy yields the best possible result of at most 19 drops when you do every 10th floor and suppose that the lowest floor to break the phone is the 99th. However, there is a strategy with only 14 drops obtained by 1st dropping the phone on the floors

$$14, \ 14 + 13 = 27, \ 27 + 12 = 39, \ 39 + 11 = 50, \ 50 + 10 = 60, \ 60 + 9 = 69, \ 69 + 8 = 77,$$

$$77 + 7 = 84, \ 84 + 6 = 90, \ 90 + 5 = 95, \ 95 + 4 = 99, \ 99 + 3 = 102, \ 102 + 2 = 104, \ 104 + 1 = 105$$

So we are actually showing we can handle 105 floors with only 14 drops. The trick is that if we get a failure along the way with the first phone, the number of checks remaining is what we have left from 14. A big question remains:

*Is 14 the best we can do - i.e. is there some really really clever scheme that guarantees at most only 13 drops?*

Another great question is what could you do with 3 phones and a 1000 floors?
TEACHER’S FLIP IT

It was not too hard to find a sequence of 5 flips to turn all the heads to tails. Therefore, the question becomes - can it be done in 4 moves or can we prove this to be impossible? Several key features of the question were realized:

- The following grid shows how many coins are controlled by each move:

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<thead>
<tr>
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<th>3</th>
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<tbody>
<tr>
<td>4</td>
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<td>3</td>
<td>4</td>
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</tbody>
</table>

- A sequence of 4 moves which accomplishes a reversal must make a total number of flips which equals 9 plus an even number. In other words, an odd number > 9. Of course a sequence of 4 moves must do more than this but it has to accomplish at least this to have any chance of being a solution.

- Possible collections of allowed moves are as follows: (classified by the number if 3 moves among the 4).
  - Three 3s and a 4.
  - Two 3s, a 5 and a 4.
  - One 3 one 4 and two 5s.
  - 3 4s and a 5.
  - One 4 and 3 5s.

- Certainly it would be possible to eliminate some of these choices based on other considerations. For example, in the case of one 4 and 3 5s we note that since only the center move creates a 5 two of these would occur in succession and thus result in no net improvement. Thus if there were a solution of the last type then there would also be a solution with only two moves and we can certainly rule that out.

Finally we ask, what happens in a $4 \times 4$, $5 \times 5$, or $6 \times 6$ grid?