

Conjecturing with the Calculator¹

1. In a finite group $G = \{g_1, g_2, \dots, g_n\}$ what can you say about

$$\prod_{i=1}^n g_i?$$

2. What happens in an abelian group? Use this to motivate a conjecture for all groups. Can you prove your conjecture for a class of groups or in general?

3. What happens when the group has odd order?

4. Does the collection of all elements of the form described in 1 give a subgroup of G ?

5. What about

$$\prod_{i=1}^n g_i^2?$$

6. What about with groups of even order when you consider products with exactly half the elements? [e.g. can you place constraints on such products to give theorems about what kind of elements will occur? Can every element be expressed in this way?]

¹This is an example activity to get students exploring and making conjectures with the software. I don't have answers to all these questions or even know if they are all interesting and tractable.