MATH301 (Introduction to Proofs)

Homework 5: (§2.7 #2,4,8,10; §2.9 #4,6,10; §2.10 #2,4,6,10)

§2.7 (1 pt each)

#2
(English) For every real number $x$, there is some natural number $n$ so that $x^n$ is nonnegative.
True. An even power of any number is nonnegative. (It would have been false if it said $x^n > 0$ instead of saying $x^n ≥ 0$.

#4
(English) For every set $X$ in the power set of the natural numbers, $X$ is a subset of the real numbers.
True. A member of the power set of natural numbers is a subset of natural numbers and the set of natural numbers is a subset of real numbers.

#8
(English) For every integer $n$, there is some subset $X$ of the natural numbers such that the cardinality of $X$ is $n$.
False. Cardinality cannot be negative, therefore for negative integers there is no such $X$.

#10
(English) There is an integer $m$ such that for every integer $n$, the sum of $n$ and 5 is $m$.
False. The statement indicates existence of a specific integer $m$ which would satisfy $m = n + 5$ regardless of what $n$ may be. In reality, $m$ will depend on $n$.

NOTE: This is not the same thing as: For every $n ∈ Z$ there is an $m ∈ Z$ such that $m = n + 5$.

§2.9 (0 correct = 0 pts, 1-2 correct = 1 pt, 3 correct = 2 pts)

#4 Let $P$ denote the set of prime numbers. $∀p ∈ P$, $∃q ∈ P$, such that $q > p$

#6 $∀ε > 0$, $∃M > 0$, such that $x > M ⇒ |f(x) - b| < ε$

#10 $\sin(x) < 0 ⇒ (∃0 ≤ x ≤ π)$
Equivalently,
$\sin(x) < 0 ⇒ ((x < 0) ∨ (π < x))$

§2.10 (1 pt each)

Note: The logical equivalence $∼ (P ⇒ Q) = P ∧ ∼ Q$ is used throughout this section.

#2 $x$ is prime and $\sqrt{x}$ is a rational number.

#4 Negation of the original statement $=∼ (∀ε > 0$, $∃δ > 0$, such that $|x - a| < δ ⇒ |f(x) - f(a)| < ε)$

$= ∃ε > 0$, such that $∼ (∃δ > 0$, $|x - a| < δ ⇒ |f(x) - f(a)| < ε)$

$= ∃ε > 0$, such that $∀δ > 0$, $∼ (|x - a| < δ ⇒ |f(x) - f(a)| < ε)$

$= ∃ε > 0$, such that $∀δ > 0$, $([x - a] < δ) ∧ (|f(x) - f(a)| ≥ ε)$
#6

\[ \sim (\exists a \in \mathbb{R}, \text{ such that } \forall x \in \mathbb{R}, a + x = x) = \forall a \in \mathbb{R}, \sim (\forall x \in \mathbb{R}, a + x = x) \]
\[ = \forall a \in \mathbb{R}, \exists x \in \mathbb{R}, \text{ such that } \sim (a + x = x) \]
\[ = \forall a \in \mathbb{R}, \exists x \in \mathbb{R}, \text{ such that } a + x \neq x \]

#10

\[ \sim ((f \text{ is a polynomial } \land \deg f \geq 2) \Rightarrow f' \neq \text{ constant}) = (f \text{ is a polynomial } \land \deg f \geq 2) \land (f' = \text{ constant}) \]